Workshop
Optimality in Algebraic Statistics

14^{th} and 15^{th} of February 2019

Department of Computer Science
Dortmund University of Applied Sciences and Arts
Room A.E.03
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Discrete Gaussian Distributions via Theta Functions

Carlos Amendola

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We study a discrete analogue of the classical multivariate Gaussian distribution. It is supported on the integer lattice and is parametrized by the Riemann theta function. Over the reals, the discrete Gaussian is characterized by the property of maximizing entropy, just as its continuous counterpart. We capitalize on the theta function representation to derive statistical properties. Throughout, we exhibit strong connections to the study of abelian varieties in algebraic geometry. This is joint work with Daniele Agostini.

Equations defining probability tree models

Eliana Duarte

Otto-von-Guericke University, Germany

Staged trees are a recent and exciting class of discrete graphical models. They are more general than discrete Bayesian Networks and provide a new framework to study these models. In this talk I will define staged tree models and give a combinatorial characterization of when these models are toric. Many examples will be given with emphasis in the toric case and its connections to the very well known decomposable models. This is joint work with Christiane Görgen (MPI Leipzig).

The 0-1-2 of local Dirac mixture moments

Alexandros Grosdos

Osnabrück University, Germany

Moments in statistics are quantities that reveal the shape of a distribution and have recently gained attention from an algebraic point of view. When they are polynomial or even rational functions of the parameters, one can define the moment ideal that is interesting both for statistical inference and for its algebraic properties. In this talk we focus on the local Dirac mixtures. The case of local order 0 is well known and corresponds to secants of Veronese varieties in algebraic geometry and Waring decompositions of symmetric polynomials. For 1st order Diracs we obtain minimal generating sets of the ideals in question. Furthermore, we apply elimination theory and Prony's method in order to do parameter estimation, and showcase our results with an application in signal processing. We finally formulate conjectures and open problems for higher local orders. This talk is based on joint work with Markus Wageringel.

Oriented Gaussoids

Thomas Kahle

Otto-von-Guericke University, Germany

An oriented gaussoid is a combinatorial structure that captures the possible signs of correlations among Gaussian random variables. We introduce this concept and present approaches to the classification and construction of oriented gaussoids.

The numerical statistical fan of a noisy experimental design

Arkadius Kalka

Dortmund University of Applied Sciences and Arts, Germany

Identifiability of models, in particular hierarchical models, is an important issue in experimental design. The theory of Groebner basis and algebraic fans has been applied to this subject. For wide classes of experimental designs the algebraic fan can be computed efficiently, because it is much smaller than the set of all identifiable maximal hierarchical models, called the statistical fan. In the case of noisy designs, e.g., when the design points themselves are observations, some models are only identifiable due to small deviations of the design. In order to avoid such unstable models, the notion of numerical algebraic fan has been developed which deploys the so-called numerical Buchberger-Möller algorithm. But the numerical algebraic fan has not been implemented yet and only subsets have been computed. In a more straightforward approach we define the numerical statistical fan. A model lies in the numerical statistical fan if the columns of its design matrix are approximatively linear independent. Several notions of approximate linear dependence for limited precision points may be applied, but we deploy the Fassino conditions which are also used in the numerical Buchberger-Möller algorithm. Using a recursive enumeration algorithm, the numerical statictical fan is computed for real data obtained from a thermal spraying process, and it is shown to be much smaller than the (exact) statistical fan.

Circuits in experimental design

Fabio Rapallo

Università del Piemonte Orientale, Italy

In the framework of factorial design, a statistical (linear) model is defined through an appropriate model matrix, which depends on the experimental runs and encodes their geometric structure. In this seminar we descuss some properties of the circuit basis of the model matrix in connection with two well-known properties of the designs, namely robustness and D-optimality. Exploiting the identification of a fraction with a binary contingency table, we define a criterion to check whether a fraction is saturated or not with respect to a given model and we generalize such a result in order to study the robustness of a fraction by inspecting its intersections with the supports of the circuits. Using some simulations, we show that the combinatorial description of a fraction with respect to the circuit basis is strictly related to the notion of D-optimal fraction and to other optimality criteria. This talk is based on joint work with Henry Wynn.

Discovering statistical equivalence classes of discrete statistical models using computer algebra

Eva Riccomagno

University of Genova, Italy

We show that representations of certain polynomials in terms of a nested factorization are intrinsically linked to labelled event trees. We give a recursive formula for the construction of such polynomials from a tree and present an algorithm in the computer algebra software CoCoA to derive all tree graphs from a polynomial. We finally use our results in applications linked to staged tree models. This is joint work with Anna Bigatti (University of Genova, Italy), Christiane Görgen and Jim Q. Smith (The University of Warwick, UK).

Rhomboid Designs for Linear Regression with Correlated Random Coefficients

Frank Röttger

Otto-von-Guericke University, Germany

We study a linear regression model $Y_i(x_i) = f(x_i)^T b_i$ on the hypercube with a linear intercept where $b_i \sim N(\beta, D)$ and all $Y_i(x_i)$ are independent, which means that there is only one observation per realisation of b_i. The parameter to be estimated is β, while D is fixed. We assume that the structure of D displays an independent linear intercept and a completely symmetric covariance matrix for the random coefficients. Through a model transformation and the introduction of rhomboid designs, we see that the Kiefer-Wolfowitz equivalence theorem implies when the optimality regions of these designs are either algebraic varieties intersected with trivial constraints or semi-algebraic sets. In fact, it shows that this distinction depends on the choice of the design points. Consequently we then discuss up to dimension 4, for which covariance matrices an optimal rhomboid design is supported either completely on the vertices of the hypercube or has support points in the interior. Furthermore, we conjecture a similar result for arbitrary dimension. This is joint work with Ulrike Graßhoff, Heinz Holling and Rainer Schwabe.

Symmetry Matters: Group Invariance in Statistics

Rainer Schwabe

Otto-von-Guericke University, Germany

The concept of invariance has a long tradition in statistics dating back at least to Pitman, 1939. Invariance or, more precisely, equivariance is related to a pair of transformations acting on the sample space of observations and on the parameter space of model parameters, respectively. These transformations are assumed to work properly together. As a simple example one may consider the simultaneous additive translation of the data and the location parameter or, slightly more complicated, the reverse scaling of the data and the effect size. In statistical estimation a third (induced) transformation may be added which acts on the response space of possible values for the estimates. If in place of single transformations whole groups of transformations are available for each of the components, then group invariance can be employed to reduce the complexity of finding solutions to statistical optimization problems.

Here we will be particularly interested in the usefulness of equivariance and group invariance in the design of statistical experiments. For linear models this approach dates back, at least, to Kiefer, 1959. Here transformations of the design region of potential experimental settings are assumed to induce (linear) transformations of the regression functions. These, however, induce transformations on the information matrix. The quality of a design is described by a design criterion, i.e. a functional on the information matrix. For equivariant design criteria design optimality is preserved by applying the original transformation on the design region to the design. In the case of group invariance invariant designs constitute an essentially complete class if the design criterion is convex and invariant. This allows the reduction of the search for an optimal design to this class which often leads to substantial simplifications.

In nonlinear situations additionally suitable transformations on the parameter space have to be added which are compatible with the transformations of the information matrix. Then equivariance can be employed to derive locally optimal designs via a canonical transformation from standardized parameter values. For group invariance optimal designs can be obtained with respect to global criteria which are defined as invariant weighted criteria or minimax criteria on an invariant parameter region.

The usefulness of this approach will be illustrated by various examples for designs in both linear and nonlinear situations.

Total positivity in structured binary distributions

Piotr Zwiernik

Pompeu Fabra University, Spain

We study distributions of binary variables that are multivariate totally positive of order 2 (MTP2) and satisfy other specific restrictions, such as, for example in Ising models in which case MTP2 corresponds to the ferromagnetic case. We apply the theory of convex exponential families of MTP2 distributions and give conditions for existence of the MLE as well as algorithms for their computation. We compare our results with the Gaussian case and identify similarities and differences. In particular we show that the MLE gives a sparse solution that offers an interesting alternative to methods for learning sparse graphical models such as graphical lasso. This is joint work with Steffen Lauritzen and Caroline Uhler