Spectral asymptotics of Robin Laplacians on polygonal domains

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Abstract

Let \( \Omega \subset \mathbb{R}^2 \) be a curvilinear polygon and \( Q_\Omega^\gamma \) be the Laplacian in \( L^2(\Omega) \), \( Q_\Omega^\gamma \psi = -\Delta \psi \), with the Robin boundary condition \( \partial_n \psi = \gamma \psi \), where \( \partial_n \) is the outer normal derivative and \( \gamma > 0 \). We are interested in the behavior of the eigenvalues of \( Q_\Omega^\gamma \) as \( \gamma \) becomes large. We prove that there exists \( N_\Omega \in \mathbb{N} \) such that the asymptotics of the \( N_\Omega \) first eigenvalues of \( Q_\Omega^\gamma \) is determined at the leading order by those of model operators associated with the vertices: the Robin Laplacians acting on the tangent sectors associated with \( \partial \Omega \). In the particular case of a polygon with straight edges the \( N_\Omega \) first eigenpairs are exponentially close to those of the model operators. Moreover, if the polygon admits only non-resonant or concave corners, we prove that, for any fixed \( j \in \mathbb{N} \), the \( N_\Omega + j \) eigenvalue \( E_{N_\Omega+j}(Q_\Omega^\gamma) \) behaves as

\[
E_{N_\Omega+j}(Q_\Omega^\gamma) = -\gamma^2 + \mu_j^D + o(1), \quad \text{as } \gamma \to +\infty,
\]

where \( \mu_j^D \) stands for the \( j \)th eigenvalue of the operator \( D_1 \oplus ... \oplus D_M \) and \( D_n \) denotes the one-dimensional Laplacian \( f \mapsto -f'' \) on \((0, l_n)\), where \( l_n \) is the length of the \( n \)th side of \( \Omega \), with the Dirichlet boundary condition.