More and more research in mathematics education points to the importance of the discursive level of language for the learning of mathematics. The presented study theoretically builds on combining an Interactional-Epistemological Perspective from mathematics education with the linguistic theory of Interactional Discourse Analysis to further investigate the role of explaining and arguing for the learning of conceptual understanding. A qualitative analysis in remediating small group courses on fractions shows that students succeed in describing observations but need a lot of support for accomplishing tasks on explaining meanings and connections.

Large quantitative studies show repeatedly that students with higher language proficiency reach higher results in mathematics tests than students with lower language proficiency (Haag, Heppt, Stanat, Kuhl, & Pant, 2013; OECD, 2007). The role of language is a large field in mathematics education research that is considered from various perspectives (Barwell et al., 2016). Research focusing on the question how to foster language learners in mathematics classrooms point to the importance of considering the discursive level of language that goes beyond word and sentence level (e.g. Barwell, 2012; Moschkovich, 2013). The presented study aims at further unfolding how the learning of conceptual knowledge (Hiebert, 1986) is connected to the discursive level of language and to what extend students need help in accomplishing the challenging task of talking about meanings and connections. For this, theoretical background on the definition of discourse practices and their role in mathematics classrooms is presented, followed by presenting the research methods. The intertwined mathematical and linguistic challenges are exemplified in the empirical part by two excerpts from a remediate small group course on fractions in grade 7. The paper ends with a discussion of these insights and an outlook on further research.

THEORETICAL BACKGROUND: THE ROLE OF DISCOURSE PRACTICES IN MATHEMATICS CLASSROOMS

There is a broad consensus in mathematics education about the importance of participation in high level mathematical and linguistic practices for all students but especially for students who are still acquiring the language of instruction (e.g. Moschkovich, 2013). The basis of this consensus is a participationist perspective on mathematics classrooms (Sfard, 2008; referring to Vygotsky, 1978) that conceptualizes learning mathematics as “a process of enculturation into mathematical practices,
including discursive practices (e.g., ways of explaining, proving, or defining mathematical concepts)” (Barwell, 2014, p. 332). In order to learn more about the role of discourse practices in mathematics classrooms, the Interactional-Epistemological Perspective (IPE) from mathematics education is enriched with the linguistic theory of Interactional Discourse Analysis (IDA) as introduced in Erath, Prediger, Heller, and Quasthoff (submitted). IPE focusses on knowledge construction and epistemic participation in the interaction of classroom microcultures (cf. Yackel & Cobb, 1996) and therefore investigates interactive processes while systematically bearing the subject in mind. Enrooted in the same ethnomethodological tradition (Garfinkel, 1967), IDA provides theories on discourse practices and discourse acquisition that facilitate a deeper understanding on the intertwinement of learning mathematics and language. For the presented study, especially the distinction between different discourse practices (e.g. explaining, arguing, narrating, reporting, describing) is used: Following IDA, discourse practices are defined as interactively co-constructed, contextualized pattern in a speech community (e.g. a mathematics classroom microculture) that are functionally oriented towards particular genres (Bergmann & Luckmann, 1995). This means that different discourse practices are distinguished by means of different ‘problems’ they solve in a speech community. For example, explanations solve the problem of conveying and constructing knowledge while arguments serve to negotiate divergent validity claims.

This way of differentiating discourse practices reveals why the practices of explaining and arguing have a special position in (mathematics) classrooms since these discourse practices meet the tasks of school: Knowledge needs to be constructed, connected, and demonstrated as well as for example different ways of thinking need to be argued. The theoretically expected dominance of explaining and arguing in classrooms can be empirically confirmed by a study on grade 5 whole class discussions in Germany (Erath et al., submitted): Explaining is the most frequently identified discourse practice in the observed mathematics classrooms followed by arguing and reporting on solution pathways. Furthermore, the interdisciplinary team points out that the linguistic demands rise with advancing in the process of knowledge construction. This means in particular, that talking about meanings, mental models, connections and other aspects of conceptual knowledge (Hiebert, 1986) is challenging for all students but especially for those with low language proficiency. But how do students with low language proficiency and teachers deal with situations in which talking about meanings and connections is necessary? Did they develop ways of bypassing the linguistic challenges? These questions are further investigated in the presented study by working on the following research questions:

Q1: Which discourse practices are used to talk about meanings and connections?

Q2: To what extent do students need support to participate in discourse practices linked with mathematical meanings and connections?
METHODOLOGY AND METHODS: QUALITATIVE ANALYSIS OF MODERATED SMALL GROUP PROCESSES

Methods of data collection. The presented data is part of the larger intervention study MESUT in which students’ conceptual understanding on fractions was fostered by means of stimulating discourse related to ‘pushed output’ and ‘relating registers’ (cf. Prediger & Wessel, submitted). 186 German grade 7 students (aged around 13 years, mainly from underprivileged urban quarters, with comparatively low language proficiency, and weak in mathematics) worked in groups of 3 to 6 students for 5 lessons together with a teacher (researchers and trained student assistants). The analysis of this paper is based on video data and written products of 9 groups.

Methods of data analysis. For this study, 4 tasks in Lesson 2 and 3 were selected for transcription. Selection criteria was that they had shown a high potential for eliciting discourse practices in a first rough analysis of the video material. The transcripts were intensively analyzed in the tradition of discourse analysis as introduced in Erath et al. (submitted): First, all episodes were identified in which an explanation or argumentation became necessary in the interaction. In a second step, these sequences were analyzed guided by the following questions: (1) which mathematical aspects were dealt with (meanings, connections, representations, procedures …)? (2) How do students verbalize their explanations and argumentations? If not successful: which discourse practice do they choose instead or do they switch to single words or half sentences? (3) How does the teacher guide and help the students in accomplishing the task? In this paper, translated and simplified versions of the transcripts and the related task (Figure 1) are printed.

Figure 1: Task on portioning more coarsely and more finely
EMPIRICAL RESULTS: EXPLAINING MEANINGS AND CONNECTIONS AS MATHEMATICAL AND LINGUISTIC CHALLENGE

Two episodes from different groups are presented as examples in this section. Previously, students were working on comparing shares in the context of downloading movies and deciding equitably who scores best in a football competition. The fraction bar (see Figure 1) is an important representation that is used systematically in the intervention. In the phase of systematizing, students are working on the task printed in Figure 1 and are asked to explain what is meant by the meaning related expressions “If I’m looking up, I’m portioning more coarsely!” and “If I’m looking down, I’m portioning more finely!” with reference to the fraction bars. Therefore, the episodes focus on a crucial and challenging moment in the process of knowledge construction: The task aims at talking about the new mental model of finding equal shares (portioning more coarsely/finely) by referring to the familiar representation. This especially means, that it is necessary to verbalize connections since the bars must be considered comparatively.

Episode 1: Lowering linguistic and mathematical demands

Group H-BP consists of four students (three male, one female) from a lower secondary school. The teacher (Te1) and the students Dennis (Den) and Rahmiye (Rah) are speaking in Episode 1, starting after the teacher asks the students to explain the expression “If I’m looking up, I’m portioning more coarsely!”:

   22 Den: I’d like to say something else
   23 Te1: W, What would you like to say?
   24 Den: The numerator has divided here, here is written eight and there four
   25 Te1: The denominator?
   26 Den: Or the denominator, no idea, down there, I don’t know what it’s called
   27 Te1: The denominator
   28 Den: Yes

In the first part of Episode 1 (#22-28), Dennis describes his observation of ‘divided’ denominators and succeeds in producing a description (#24), even though with a wrong term which is corrected by the teacher. In the second part (#29-38), the teacher builds on this observation by asking the students to connect it to the representation (#29/32/36) and thus making an explanation necessary that draws connections between denominators and fraction bars as well as between the two considered fraction bars. This seems mathematically and linguistically challenging for the students:

   29 Te1: Exactly, and what..the denominator um divided by two, right so in half, and what does it with the bar? [points to the bar in the task, 8 sec. break] whereby do I see at the bar that down here, the denominator is eight and above four
   30 Den: Because above#
   31 Rah #It doubles
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32 Te1: Yes what doubles? Explain it
33 Rah: The denominator
34 Te1: The denominator from, from top to bottom it doubles, well, and how do I see it at the bar?
35 Den: Because the pieces are larger
36 Te1: Exactly, how many pieces and this is related to your doubling or halving, how many pieces are then one piece? If I’m looking the bottom up
37 Rah: Two
38 Den: Two

Rahmiye (#31/33) rephrases the observation without reference to the representation, Dennis (#35) comparatively refers to the size of the pieces but without explicitly drawing a connection between the two fraction bars and the denominators. The teacher supports the students in their verbalizations by asking clarifying questions that can be answered by single words (#36) and therefore simplifies the task linguistically (and at the same time lowers the mathematical requirements). This, as well as making explicit that an explanation (and not a description) is needed (#32) and constantly building on students’ previous utterances, helps the students to remain active parts of the conversation. However, they do still not succeed in producing the pursued explanation. At the end (#39), it is the teacher who merges the students’ observations in an explanation and makes the connections between the size of the pieces in the fraction bar and the meaning related expression of portioning more coarsely:

39 Te1: Correct yes, two pieces, make one lar um, make one large and this means Kenan with ‘if I’m looking up I’m portioning more coarsely’ […]

On the one hand, the teacher’s explanation can be interpreted as keeping the students back from further attempts and hence learning opportunities regarding explaining meanings and connections. On the other hand, it can be seen as supportive since it can serve as a model for an explanation that also shows the expectations of the teacher.

**Episode 2: Preserving linguistic and mathematical demands**

Group M-SP consists of four students (two male, two female) from a middle secondary school. The teacher (Te2) and the students Cemil (Cem), Lorik (Lor), and Lisa (Lis) are speaking in Episode 2, starting after the teacher asks the students to explain the meaning related expressions printed in the task (see Figure 1):

74 Cem: fine is, well, small and more coarsely is large
75 Lor: Small but fine
76 Te2: [To Cemil] How do you mean that exactly?
77 Cem: Like just#
78 Te2: #Can one describe that again differently?
79 Cem: Yeeeeeesss, difficult [smiles]
In the first part of Episode 2 (#74-79) Cemil renames more finely and coarsely as small and large which the teacher asks him to explain in more detail, probably because small and large do not refer to comparatives (as smaller and larger). Also the students in this group struggle with verbalizing the required explanation as explicitly stated by Cemil in #79 while succeeding in describing observations regarding the printed shares: The second part of Episode 2 (#80-83) deals with Lisa’s observation that the same share can be expressed differently:

80 Lis: [raises her hand] ..But I know how it is correct, because here the share is written [points to the task] so 6/8 and with the others it’s not written, it was then portioned by um different fractions, so ¾

81 Te2: Mhm so I can express the shares differently, right?

82 Lis: Yes

This is marked as correct by the teacher who nevertheless navigates back to Cemils utterance from the beginning. The teacher in this group does not lower the linguistic and mathematical demands, but continuously asks clarifying questions that still require answers longer than half sentences (#76/78/83).

83 Te2: That’s [true] in any case, and what was that with the more finely and more coarsely again?#

84 Cem: #Yes for example this one here, um, which is larger [points to a piece in the bar of 4th] and that the finer, that two fit in there

85 Te2: Ah okay! So this one piece becomes two, if I portion more finely and the other way round, if I make one out of two?

In the third part of Episode 2 (#83-85), Cemil succeeds in producing an explanation that compares the two fraction bars and verbalizes the connection between the sizes of the pieces backed by deictical means. The teacher amplifies his explanation by connecting it to the expressions more coarsely/finely given by the material.

**DISCUSSION AND OUTLOOK**

The two episodes are examples of phenomena that can be recurrently observed across the five sessions of the intervention as well as across different groups. Talking about meanings and connections in mathematics is (from a subject perspective as well as from a linguistic perspective) challenging for the students. The latter is even addressed explicitly by the students: Dennis (Episode 1, #26) struggles on word level, Cemil (Episode 2, #79) admits the difficulty in enhancing his utterance towards an explanation. Furthermore, both episodes show how the teachers try to support their students in order to help them verbalize their ideas as well as guide them towards the mathematical learning goal. The analysis thus shows once again how language and subject matter cannot be seen as separated. Furthermore, Episode 1 demonstrates how students learn vocabulary while participating in discourse practices.

Research question Q1 on which discourse practices are used for discussing connections and meanings in mathematics classrooms can be answered by pointing out that the
mathematical learning goal of understanding the meaning related expression on finding equal shares and connecting it to the familiar representation was only addressed by explanations in the presented episodes. The offered descriptions were used as starting point by the teacher in Episode 1 and were marginally approved in Episode 2, but these discourse practices did not succeed in verbalizing the connections between the representations and between the fraction bars and the meaning related expression. Thus, the theoretical and empirical importance of explaining and arguing (see above) is affirmed and shown on a micro level in this study.

The data shows that students need help to participate in the challenging practices of explaining and arguing in mathematics (research question Q2). Both teachers have to invest time and several moves in order to end the sequence with an explanation that meets the task. The two presented teachers deal differently with this challenge and of course adapted to their group and the ongoing interaction. From the data, we cannot conclude if staying on the initial high mathematical and linguistic level (e.g. also recommended by Moschkovich, 2013) would have also led to students’ explanations in Episode 1. However, since in both episodes, students offer descriptions, it can be assumed that it might be effective to find a way of keeping the linguistic and mathematical demands (i.e. staying on a discursive level) and at the same time jointly converting students’ descriptions to explanations. The question how this might work must stay unanswered here, but points to possible further research.

So far, we know little about teacher moves that support students on the level of discourse practices (cf. Erath, submitted). More research is needed to identify ways of enabling all students to actively participate in demanding discursive and at the same time mathematical practices since they are closely linked to important learning opportunities. Given the identified connection between explanations and argumentations and working on conceptual knowledge, these practices need to be considered in particular. This is further supported by first quantitative results (based on 9 groups) from the project MESUT (Nienhoff, 2017): The higher the share of explanation and argumentation in the total time of group discussion (excluding individual seatwork or working in pairs) the higher the gains in mathematical achievement. At the moment, this analysis is expanded to more groups. Nevertheless, it hints at a first quantitative support of the qualitative observation that including all students (especially the vulnerable) in these challenging discourse practices in particular means to offer important mathematical learning opportunities.

Remark: The research project MESUT – Developing conceptual understanding by language support. Differential effects of language- and content-integrated approaches – is funded by the German Research Foundation (DFG; grant PR 662/14-1 to S. Prediger). The author conducts it with Susanne Prediger and Lena Wessel.

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