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Influence of Variable Thermophysical Properties on Nanofluid Bioconvection along a Vertical Isothermal Cone

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1 Abstract

The prime purpose of this analysis is to investigate the effect of variable thermophysical properties of nanofluid on bioconvection boundary layer flow past a uniformly heated vertical cone. The governing equations with associated boundary conditions for this phenomenon are cast into a non-dimensional form via continuous transformation, which are then solved by using the implicit finite difference method. How does the phenomena of heat transfer is effected due to the temperature-dependent thermophysical properties of the fluid is the question investigated in this manuscript. Comprehensive numerical computations are carried out for wide range of the parameters that are describing the flow characteristics. The influence of the governing physical parameters on the distribution of velocity and temperature profiles, wall skin friction and local rate of heat transfer is explored. Tabular comparison with some special cases of published data is done and good agreement is found. It is recorded that, the variable thermophysical properties of the fluid are more likely to promote the rate of heat transfer, Q_w as compared to fluid having constant properties.

Keywords: Nanofluid, Bioconvection, Vertical cone, Gyrotactic microorganisms, Variable Thermophysical properties.

2 Introduction

Bioconvection is phenomenon in the area of fluid mechanics, that deals with the suspensions of self-propelled microorganisms. It is worthy to mention here that, bioconevction is different from typical multi-phase flows, where particles are not self-propelled; they are only carried by the flow. Bioconvection originates due to instability in density stratification, which is created by directional swimming of microorganisms that are heavier than their surrounding fluid (i.e, water). These self-propelled motile microorganisms tends to concentrate near the upper portion of the fluid layer, and this accumulation makes the upper layer much denser than the lower region and ultimately produce instability, which results in generating the various flow patterns into the system (for details see Refs. [1]-[6]). Bioconvection has numerous applications in biological and bio-microsystems, for instance, enzyme biosensors and biotechnology due to the mass transport enhancement and mixing, which are important issues in many micro-systems. Another potential application of bioconvection theory is microbial-enhanced oil recovery, when microorganisms and nutrients are injected

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in oil-bearing layers to correct permeability variation. Besides, the property of directional motion of motile microorganisms may be used for the concentration of cells, separation of dead and living cells, purification of cultures, or separation of various sub-populations ([7]-[9]). However, bioconvection systems may be classified depending upon the directional movement of various species of microorganisms, but, they usually swim in upward direction (having larger density than the base fluid). For instance, i) chemotaxis or oxytactic type of microorganisms swims upwardly due to the gradient of oxygen concentration, as they require certain amount of oxygen concentration to be active, ii) gyrotactic microorganisms whose swimming's direction is determined by making a balance in viscous and gravitational torques, and iii) geotactic microorganisms swim against the gravitational effects ([10]-[11]). In addition, the concept of nanofluid bioconvection has also wide circle of applications, for instance, nanomaterial processing, automotive coolants, sterilization process of medical suspensions, polymer coating and intelligent building design. The idea of nanofluid bioconvection was first introduced by Kuznetsov ([12]-[13]), and later on, due to characteristics of nanofluids to promote the rate of heat transfer, numerous authors and analysts investigated the interaction of nanofluid with bioconvection (see Refs. [14]-[20]).

The class of problems involving the variable thermophysical properties of the fluid has attracted numerous researchers and experimentalists due to their significant applications in viscometery, food processing, instrumentation, tribology, lubrication and many others processes of engineering occurring at high temperatures. In order to predict the flow behavior accurately, it is important to take into the account the temperature-dependent thermophysical properties of the fluid. Until date, numerous researchers addressed the variable thermophysical properties of the fluids and reported the interesting facts on fluid flow and heat transfer. In this regard, the analysis of the variable thermophysical properties of the fluid for laminar natural convection past an isothermal vertical wall has been reported by Sparrow and Gregg [21]. After that, Brown [22] investigated the influence of the volumetric expansion coefficient on heat transfer rate for the problem of natural convection. In addition, the validity of the Boussinesq approximation for liquids and gases was investigated by Gray and Giogini [23]. In this paper, the authors also suggested a method for analyzing natural convective flows for the fluids with variable properties. Besides, Clausing and Kempka [24] investigated the effect of variable properties on experimental basis and reported that, the rate of heat transfer Nu will be a function of Ra, only with reference temperature, T_f, which is taken as the average temperature in boundary layer regime. Furthermore, the analysis of i) instability of laminar natural flow and iii) transition from laminar to turbulent flow had been presented by Gebhart [25] and also have been summarized in a textbook [26]. A detailed analysis of effect of variable thermophysical properties on laminar free convection of gas has also been presented in [27].

It is found that the problem of nanofluid bioconvection with variable thermophysical properties along a vertical cone has not been treated in the literature. Motivated by the previous works and possible applications, present study is conducted to report the influence of temperature dependent thermophysical properties of the base fluid on nanofluid bioconvection flow. In this study, it is assumed that, i) the gyrotactic microorganisms are self-propelled, ii) the nanoparticles just move due to Brownian motion and thermophoresis and are carried by the flow of the base fluid, iii) nanoparticles in base fluid does not effect the directional motion of microorganisms. Besides, assumptions have been made that the viscosity $\mu \approx T^{n_\mu}$ and the thermal conductivity $\kappa \approx T^{n_\lambda}$ and the density is reciprocal of the absolute temperature. On the basis of these physical assumption, the interaction of combined effect of microorganisms, nanoparticles and variable thermophysical properties presents an interesting fluid dynamics problem. Primitive variable formulation is incorporate for converting the boundary layer equations into a suitable form, and then, coupled nonlinear equations are solved iteratively via two-point finite difference method. The computational data is presented graphically in the form of skin friction coefficient, heat transfer rate, velocity and temperature profiles, streamlines and isotherms by varying several controlling parameters. We believe that the present results are a new addition to the open literature on nanofluid bioconvection.

3 Problem Formulation

Consideration have been given to the bioconvection boundary layer flow of nanofluid along a vertical isothermal heated cone. It is assumed that the water-based nanofluid is having temperature-dependent thermophysical properties and contains concentration of gyrotactic microorganisms. The surface of the cone is heated with T_w, such that, $T_w > T_\infty$, where T_∞ represent the ambient temperature. The schematic of the plate geometry and the associated coordinate system is shown in Fig. 1. By taking into account the assumptions for boundary layer flow of nanofluid bioconvection and variable thermophysical properties presented in (see Refs. [13], [15], [27]), the mathematical model is established as:

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} + \frac{1}{\rho}\left(u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y}\right) = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\nu}{\nu_f}\left[\frac{\partial^2 u}{\partial y^2} + \frac{1}{\mu}\frac{\partial \mu}{\partial y}\frac{\partial u}{\partial y}\right] + \theta - NrC - RbN \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\nu}{\nu_f}\left[\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\kappa}\frac{\partial \kappa}{\partial y}\frac{\partial \theta}{\partial y}\right] + \frac{N_B}{Ln}\frac{\partial C}{\partial y}\frac{\partial \theta}{\partial y} + \frac{N_A N_B}{Ln}\left(\frac{\partial \theta}{\partial y}\right)^2 \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{1}{Ln}\frac{\partial^2 C}{\partial y^2} + \frac{N_A}{Ln}\frac{\partial^2 \theta}{\partial y^2} \tag{5}$$

$$u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y} + \frac{\partial}{\partial x}\left[(N+\Omega)\,u\right] + \frac{\partial}{\partial y}\left[(N+\Omega)\,v\right] + \frac{Pe}{Lb}\frac{\partial}{\partial y}\left[(N+\Omega)\frac{\partial C}{\partial y}\right] = \frac{1}{Lb}\frac{\partial^2 N}{\partial y^2} \tag{6}$$

subject to the boundary conditions:

$$\begin{aligned} &u(x,0) = v(x,0) = 0, \quad \theta(x,0) = C(x,0) = N(x,0) = 1, \\ &u(x,\infty) = v(x,\infty) = 0, \theta(x,\infty) = 0, C(x,\infty) = 0, N(x,\infty) = 0 \end{aligned} \tag{7}$$

where the transformation used to convert the system in dimensionless form are:

$$x = \frac{\bar{x}}{L}, \quad y = \frac{\bar{y}}{L} Gr_L^{1/4}, \quad \bar{u} = \frac{\nu_f u}{L} Gr_L^{1/2}, \quad \bar{v} = \frac{\nu_f v}{L} Gr_L^{1/4}, \quad r = \frac{\bar{r}}{L}, \quad \bar{p} = \frac{Gr_L \rho_f \nu_f^2 p}{L^2},$$
$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}, \quad N = \frac{n - n_\infty}{n_w - n_\infty}, \quad \lambda = \frac{T_w - T_\infty}{T_\infty}, \quad Gr_L = \frac{\lambda g \cos\omega L^3}{\nu_f^2},$$
$$Nr = \frac{(\rho_p - \rho_f)(\phi_w - \phi_\infty)}{\rho_f \lambda}, \quad \Pr = \frac{\nu}{\alpha} \quad Rb = \frac{(\rho_m - \rho_f)\gamma(n_w - n_\infty)}{\rho_f \lambda}, \quad N_A = \frac{D_T(T_w - T_\infty)}{D_B T_\infty(\phi_w - \phi_\infty)},$$
$$Lb = \frac{\nu_f}{D_{mo}}, \quad N_B = \tau(\phi_w - \phi_\infty), \quad \Omega = \frac{n_\infty}{(n_w - n_\infty)}, \quad Pe = \frac{bW_{mo}}{D_{mo}}, \quad Ln = \frac{\nu_f}{D_B} \tag{8}$$

In the above system of equations, (u, v) are (x, y) components of the velocity field, p the pressure, θ the dimensionless temperature, C the concentration of nanoparticle and N the concentration of microorganisms in boundary layer region. Here r is local radius of even surface of the cone and $r = x \sin\omega$, where ω being the half cone angle. Furthermore, modified diffusivity ratio and particle-density increment are denoted by N_A and N_B, respectively. In the above expressions, Gr_L, Pr, Nr and Ln are respectively the Grashof number, Prandtl number, buoyancy ratio parameter and nanoparticle Lewis number. The bioconvection Rayleigh number, Lewis number and Pèclet number are symbolized by Rb, Lb and Pe respectively. In addition, ϕ_w represents the nanoparticle volume fraction, n_w the density of microorganisms whereas the nanoparticle volume fraction and density of the microorganisms are denoted as ϕ_∞ and n_∞, respectively. Moreover, Ω is the microorganisms concentration difference parameter, ν_f is the kinematic viscosity of the suspension of nanofluid and microorganisms, ρ the density of base fluid, ρ_p the density of the nanoparticles, ρ_m the microorganisms density, $(\rho_m - \rho_f)$ the density difference between a cell and a base fluid, g the gravitational acceleration, β the volumetric expansion coefficient of base fluid, γ the average volume of a microorganism, α the thermal conductivity of the nanofluid, $\tau = (\rho c)_p / (\rho c)_f$ the ratio of heat capacity of nanofluid to the heat capacity of the base fluid, D_B the Brownian diffusion coefficient and D_T the thermophoretic diffusion coefficient. In addition, D_{m0} is the diffusivity of microorganisms, b the chemotaxis constant and W_{m0} the maximum cell swimming speed with the property bW_{m0}=constant.

There are very few forms of temperature-dependent thermophysical properties of the fluids available in the literature. Among them, we have considered that ones which are appropriate for the class of gases introduced by ([27], [28]) and mathematically can be expressed as follows:

$$\mu/\mu_f = (T/T_\infty)^{n_\mu}, \quad \kappa/\kappa_f = (T/T_\infty)^{n_\lambda}, \quad \rho/\rho_f = (T/T_\infty)^{-1}, \tag{9}$$

where, μ_f, ρ_f and κ_f are respectively, the dynamic viscosity, the density and the thermal conductivity of the carrier fluid in the free-stream region. Furthermore, the values of n_μ and n_λ are taken from the analysis of Hisenrath *et al.* [28], which is based on experimental values of μ and κ for various diatomic and monoatomic gases, and also for water vapours and air.

Now, in order to establish the numerical solutions of the coupled Eqs. (1-6) along with the boundary conditions in Eq. (7), we switched into another system of equations with the another set of continuous transformations as given below:

$$y = Yx^{\frac{1}{4}}, \quad u = Ux^{\frac{1}{2}}, \quad v = Vx^{\frac{-1}{4}}, \quad x = X, \quad C(x,y) = C(X,Y), \quad N(x,y) = N(X,Y),$$
$$\theta(x,y) = \theta(X,Y), \quad r = X\sin\omega, \quad \frac{\nu_f}{\nu} = (1+\lambda\theta)^{-(1+n_\mu)} \tag{10}$$

The above boundary layer Eqs. can be further written in the non-conserved form as follows:

$$\frac{3}{2}U + X\frac{\partial U}{\partial X} - \frac{Y}{4}\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} - \frac{\lambda}{(1+\lambda\theta)}\left(XU\frac{\partial\theta}{\partial X} + \left(V - \frac{1}{4}YU\right)\frac{\partial\theta}{\partial Y}\right) = 0 \tag{11}$$

$$(1+\lambda\theta)^{-(n_\mu+1)}\left(\frac{1}{2}U^2 + XU\frac{\partial U}{\partial X} + \left(V - \frac{1}{4}YU\right)\frac{\partial U}{\partial Y} - \theta + NrC + RbN\right) = \frac{\partial^2 U}{\partial Y^2} + \frac{\lambda n_\mu}{(1+\lambda\theta)}\frac{\partial\theta}{\partial Y}\frac{\partial U}{\partial Y} \tag{12}$$

$$(1+\lambda\theta)^{-(n_\mu+1)}\left[XU\frac{\partial\theta}{\partial X} + \left(V - \frac{1}{4}YU\right)\frac{\partial\theta}{\partial Y}\right] = \frac{1}{Pr}\left[\frac{\partial^2\theta}{\partial Y^2} + \frac{\lambda n_\lambda}{(1+_\lambda\theta)}\left(\frac{\partial\theta}{\partial Y}\right)^2\right] + (1+\lambda\theta)^{-(n_\mu+1)}\frac{N_B}{Ln}\left(\frac{\partial C}{\partial Y}\frac{\partial\theta}{\partial Y} + N_A\left(\frac{\partial\theta}{\partial Y}\right)^2\right) \tag{13}$$

$$UX\frac{\partial C}{\partial X} + \frac{\partial C}{\partial Y}\left(V - \frac{1}{4}YU\right) = \frac{1}{Ln}\left(\frac{\partial^2 C}{\partial Y^2} + N_A\frac{\partial^2\theta}{\partial Y^2}\right) \tag{14}$$

$$2XU\frac{\partial N}{\partial X} + 2\left(V - \frac{1}{4}YU\right)\frac{\partial N}{\partial Y} - (N+\Omega)\left(U - \frac{\lambda}{(1+\lambda\theta)}\left(XU\frac{\partial\theta}{\partial X} + \left(V - \frac{1}{4}YU\right)\frac{\partial\theta}{\partial Y}\right)\right) = \frac{1}{Lb}\left(\frac{\partial^2 N}{\partial Y^2} - Pe\left((N+\Omega)\frac{\partial^2 C}{\partial Y^2} + \frac{\partial N}{\partial Y}\frac{\partial C}{\partial Y}\right)\right) \tag{15}$$

along with the transformed boundary conditions:

$$U(X,0) = V(X,0) = 0, \quad \theta(X,0) = C(X,0) = N(X,0) = 1,$$
$$U(X,\infty) = V(X,\infty) = 0, \theta(X,\infty) = C(X,\infty) = N(X,\infty) = 0 \tag{16}$$

These equations are obtained by exploiting the following mathematical expressions:

$$\frac{1}{\rho}\frac{\partial\rho}{\partial Y} = -\frac{\lambda}{(1+\lambda\theta)}\frac{\partial\theta}{\partial Y}, \quad \frac{1}{\rho}\frac{\partial\rho}{\partial X} = -\frac{\lambda}{(1+\lambda\theta)}\frac{\partial\theta}{\partial X}, \quad \frac{1}{\mu}\frac{\partial\mu}{\partial Y} = \frac{\lambda n_\mu}{(1+\lambda\theta)}\frac{\partial\theta}{\partial Y},$$
$$\frac{1}{\kappa}\frac{\partial\kappa}{\partial Y} = \frac{\lambda n_\lambda}{(1+\lambda\theta)}\frac{\partial\theta}{\partial Y}, \tag{17}$$

In order to numerically solve the non-linear system of partial differential equations, (11)-(16), implicit finite difference method is employed. The discretization procedure and numerical scheme is carried out by considering the details given in [29]. After determining all the unknown of the system, the dimensionless expressions for the physical quantities of interest like skin friction coefficient, τ_w and heat transfer rate Q_w are obtained as:

$$\tau_w = C_f\left(\frac{Gr^{-3}}{X}\right)^{1/4} = (1+\lambda)^{-(1+n_\mu)}\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$$
$$Q_w = Nu\left(\frac{Gr}{X}\right)^{-1/4} = -(1+\lambda)^{n_\lambda}\left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \tag{18}$$

In the upcoming section, the obtained results are graphed and discussed.

4 Results and Discussion

The prime interest of present study is to present the detailed numerical solutions for bio-convection boundary layer flow of nanofluid along a vertical isothermal cone. We have performed two-dimensional simulations for the mathematical model presented in terms of primitive variables in Eqs. (11)-(16) from the two-point implicit finite difference method. In order to investigate the overall effectiveness of variable thermophycial properties in bioconvection, computations are made by considering water (i.e., ($\mathrm{Pr} = 7.0, n_\mu = 1.04, n_\lambda = 1.185$) as a participating fluid. In order to ensure the accuracy of our scheme and numerical results, comparison is also being made with already available published data. It should be noted that the numerical results obtained herein, reduce to those reported by Shang and Wang ([27]) provided that we performed simulations in absence of nanofluid bioconvection along a vertical wavy wall. In addition, the study of Siddiqa *et al.* [30] can also be deducted from this analysis, when we take nanofluid bioconvection parameters equal to zero. The results are compared in tabular form, for rate of heat transfer, and are entered in Tab. 1. Here, the calculated results of $-\left(\partial\Theta/\partial Y\right)_{Y=0}$ for different fluids agrees well with those reported in ([27], [30]).

Table 1: The comparison between calculated values of $-\left(\partial\Theta/\partial Y\right)_{Y=0}$ and those reported in Refs. ([27], [30]) .

T_w/T_∞	H_2 $\mathrm{Pr} = 0.68$ $n_\mu = 0.68$ $n_\lambda = 0.8$			Air $\mathrm{Pr} = 0.7$ $n_\mu = 0.68$ $n_\lambda = 0.81$		
	Ref. [27]	Ref. [30]	Present	Ref. [27]	Ref. [30]	Present
3	0.1974	0.19736	0.19727	0.1987	0.19868	0.19859
5/2	0.2300	0.22995	0.22985	0.2316	0.23160	0.23150
2	0.2772	0.27719	0.27707	0.2794	0.27938	0.27926
3/2	0.3526	0.35254	0.35242	0.3557	0.35568	0.35557
5/4	0.4105	0.41045	0.41306	0.4188	0.41878	0.41432
1/2	0.8774	0.87793	0.87879	0.8898	0.89027	0.89113

Figure 2 is plotted to report the variations in skin friction coefficient, τ_w and rate of heat transfer Q_w, for different values of parameter λ. It is important to mention here that, $\lambda = 0.0$ recovers the solutions for nanofluid bioconvection model with constant properties. It can be seen from Fig. 2(a), that the skin friction coefficient is sufficiently reduces by owing an increase in the values of parameter λ. Here, τ_w shows its maximum values for the fluid having constant thermophysical properties (i.e, $\lambda = 0.0$), and becomes very low when the fluid undergoes temperature-dependent effect of these properties (i.e, $\lambda \neq 0.0$). While on the other hand, reverse behavior is recorded in the plots of rate of heat transfer in Fig. 2(b), as Q_w is boosted up for non-zero values of parameter λ. This is expected because, large values of λ participates in increasing the wall heating effect, which results in accelerating the fluid flow adjacent to the heated surface. Consequently, the temperature gradient increases and the rate of heat transport is intensified in the boundary layer regime.

The axial distribution of the skin friction coefficient, τ_w and the rate of heat transfer,

Q_w are represented in Fig. 3, for water with i) uniform thermophysical properties ($n_\mu = n_\lambda = 0.0$) and ii) variable thermophysical properties ($n_\mu = 1.04, n_\lambda = 1.185$). It can be visualized from Fig. 3(a), that the non-zero values of parameters n_μ and n_λ has a retarding effect on skin friction coefficient. This may happen, because water become less viscous and less dense due to the temperature variations, and consequently, the frictional forces becomes less influential in boundary-layer regime. On contrary, the rate of heat transfer is too low for the water having constant thermophysical properties, (i.e, $n_\mu = n_\lambda = 0.0$). The whole convective regime is hotter for the water with non-uniform thermophysical properties and it leads to increase the temperature gradient in the thermal boundary-layer region. Thus, it can be concluded that, the fluids having temperature-dependent thermophysical properties are more likely to promote Q_w than the fluid with uniform properties.

The influence of modified diffusivity ratio parameter, N_A and the particle-density increment parameter, N_B on skin friction coefficient and rate of heat transfer, is shown in Fig. 4. Interestingly, it can be seen from Fig. 4(a), that the skin friction coefficient depicts an insensitive behavior for overall range of parameters N_A and N_B. But, modified diffusivity ratio and particle-density increment parameters has a notable effect on the graphs of rate of heat transfer (see Fig. 4(b)). It is recorded that the rate of heat transfer is maximum when both the parameters are taken as zero. The non-zero values of modified diffusivity parameter, N_A has a tendency to decrease the rate of heat transfer at the surface of vertical cone. This is expected, due to the fact that the rate of viscous diffusion get stronger due to the increasing values of N_A, that leads to remarkable reduction in Q_w. Particularly, modified diffusivity ratio parameter is more influential in reducing the rate of heat transfer together with non-zero values of particle-density increment parameter, N_B. Similarly, the particle-density increment parameter, N_B acts as retarding factor on the rate of heat transfer. The non-zero values of N_B results in more collisions in the fluid particles, that produces more heat and consequently, the rate of heat transfer at the surface is reduced. Thus, the heat transfer characteristics of the fluid can be remarkably effected by the suspension of nanophase particles in cooling or heating fluids.

In order to illustrate the influence of buoyancy ratio parameter, Nr, on skin friction coefficient and rate of heat transfer, Fig. 5 is plotted. It is important to mention here that, parameter Nr is responsible to couple the momentum and the nanoparticle concentration equations (see Eqs. (12) and (14)). The buoyancy influences the velocity and temperature gradient; therefore, the parameter Nr has a notable effect on skin friction coefficient and rate of heat transfer. As, it can be noted from Fig. 5, that both physical quantities τ_w and Q_w undergoes a considerable reduction by owing an increase in the values of Nr from 0.0 to 0.5. This may happens because, for large values of buoyancy ratio parameter, the base fluid (water) loses the kinetic as well as thermal energy by the inter-collision of the fluid's molecules, which results in considerable decline for τ_w and Q_w near the flow axis.

Figure 6 is representing the fluctuations in: velocity and temperature profiles, which are brought by varying the values of parameter, λ, for water based nanofluid. It is observed in Fig. 6(a), that the velocity is retarded and its peak moves away from the interface by increasing the values of λ. However, it is interesting to see that velocity profiles for $\lambda = 0.0$ quickly acquires the asymptotical state in the momentum boundary layer region. Thus, non-zeros values of λ are responsible to delay the velocity profile to reach its asymptotic behavior. While on the other hand, Fig. 6(b) is depicting that the temperature profiles grows sufficiently when λ increases from 0.0 to 5.0. This is to be expected because large values of λ accelerates the fluid flow and increases the temperature as already mentioned in discussion of Fig. 2.

Figure 7 deals with the influence of variable thermophysical properties parameters, n_μ and n_λ, on the velocity and temperature profiles. For comparison, graphs are also presented with uniform thermophysical properties (i.e, $n_\mu = n_\lambda = 0.0$). It can be visualize from Fig. 7(a), that the effect of temperature-dependent thermophysical properties is to reduce the velocity profile for water based nanofluid. The velocity profile achieve the limiting behavior more rapidly in the free stream regime, for the water with constant properties. But, the temperature profile depicts the reverse behavior for the water having variable thermophysical properties (see Fig. 7(b)). Thus, non-zero values of n_μ and n_λ acts like a supportive driving force that accelerates the temperature of the water within the boundary layer region.

The effect of variable thermophysical properties parameters on the formation of streamlines and isotherms is presented in Fig. 8. When the values of thermophysical properties changes from ($n_\mu = n_\lambda = 0.0$) to ($n_\mu = 1.04, n_\lambda = 1.185$), the flow flux get stronger in comparison with the case of uniform thermophysical properties. This is expected because, when the properties of the water are taken as temperature-dependent, the molecular motion of the water increases and ultimately the flux of the flow gets augmented. Specifically, this effect is more notable on the graph of isotherms plotted in Fig. 8(b). It is observed that the fluid temperature increases sufficiently which causes the thickness of thermal boundary layer to grow.

5 Conclusion

This paper aims to compute the influence of temperature-dependent thermophysical properties on nanofluid bioconvection along the geometry of isothermally heated vertical cone. The major focus of this study is to visualize the flow characteristics of water based nanofluid when the thermophysical properties changes from uniform to variable. The system of coupled non-linear boundary layer equations is solved iteratively, via two-point finite difference method along with tri-diagonal solver. The pertinent results from this study are presented in graphical form and discussed quantitatively with respect to the variation in the values of physical parameters. Influence of various emerging parameters are explored by expressing their relevance on skin friction coefficient and rate of heat transfer. Velocity and temperature distributions are plotted and visualized as well through streamlines and isotherms. From this analysis, it is recorded that variable thermophysical properties parameters, λ, n_μ and n_λ extensively boost up the rate of heat transfer, whereas, modified diffusivity ratio parameter N_A and particle-density increment parameter N_B are retarding factors for Q_w in the boundary layer region.

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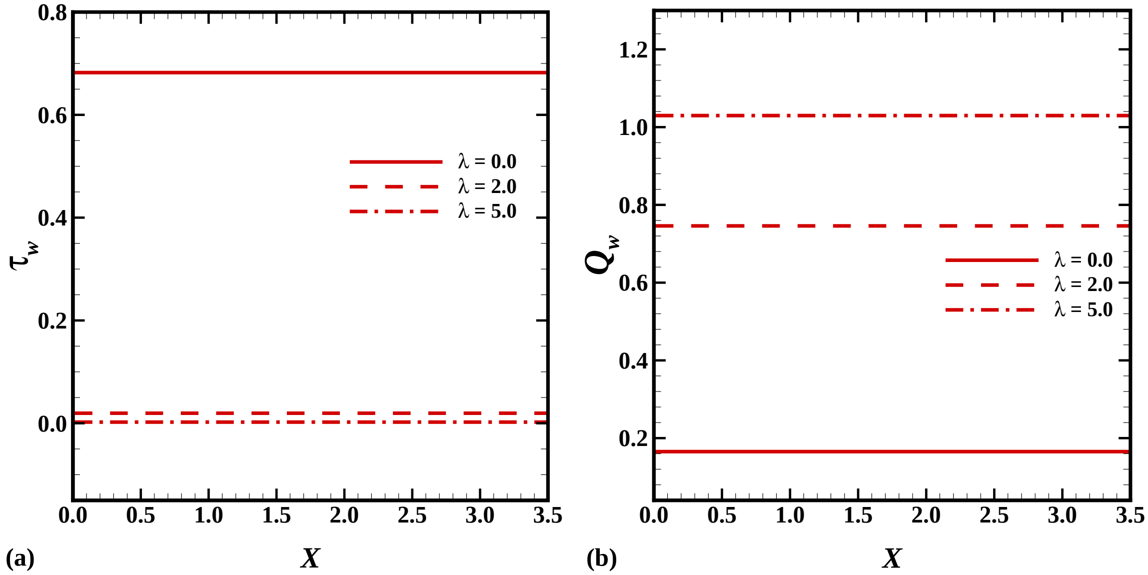


Fig. 2(a) Skin friction and (b) Rate of heat transfer coefficients for $\lambda = 0.0, 2.0, 5.0$ while Pr $= 7.0$, $n_\mu = 1.04$, $n_\lambda = 1.185$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Nr = Rb = 0.1$, $N_A = N_B = 2.0$ and $\Omega = 0.1$.

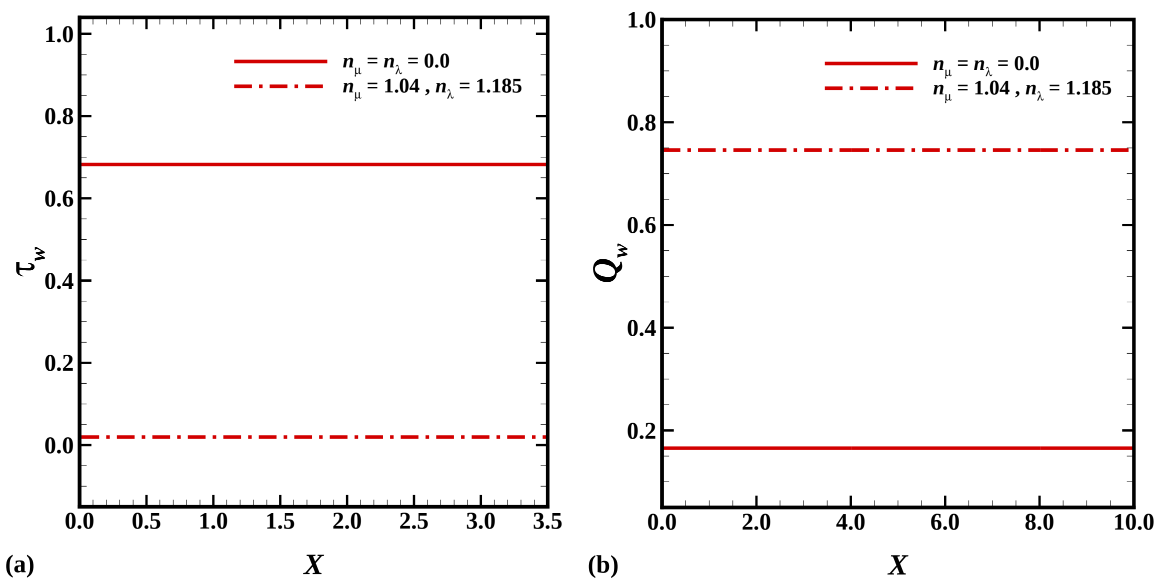


Fig. 3(a) Skin friction and (b) Rate of heat transfer coefficients for $n_\mu = 0.0, 1.04$, $n_\lambda = 0.0, 1.185$, $\lambda = 0.0, 2.0$ while Pr $= 7.0$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Nr = Rb = 0.1$, $N_A = N_B = 2.0$ and $\Omega = 0.1$.

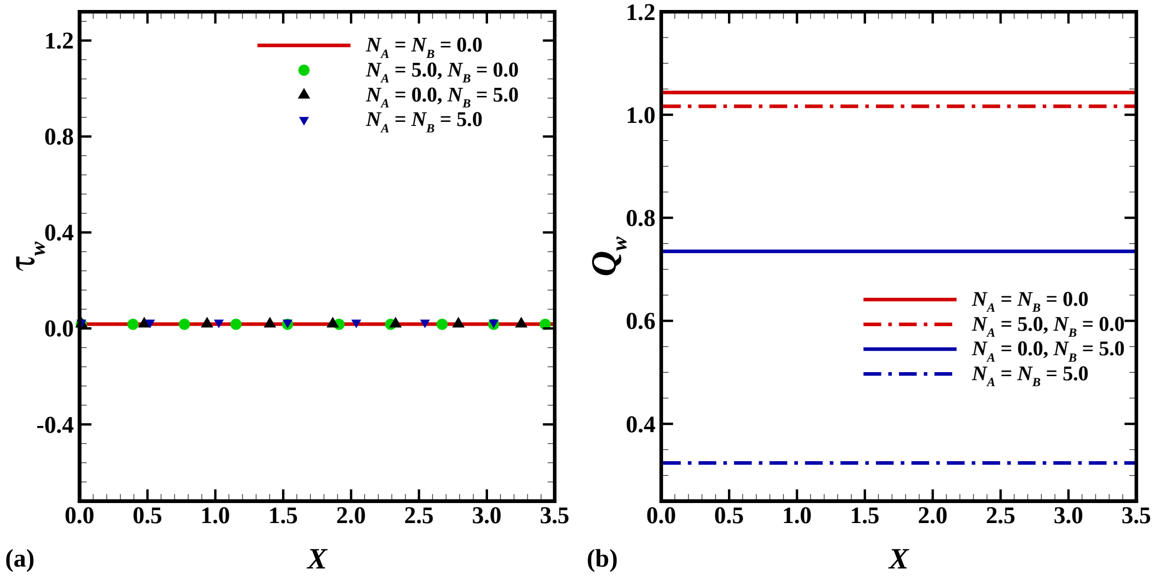


Fig. 4(a) Skin friction and (b) Rate of heat transfer coefficients for $N_A = 0.0, 5.0$, $N_B = 0.0, 5.0$ **while** $\mathbf{Pr} = 7.0$, $n_\mu = 1.04$, $n_\lambda = 1.185$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Nr = Rb = 0.1$, $\lambda = 2.0$ **and** $\Omega = 0.1$.

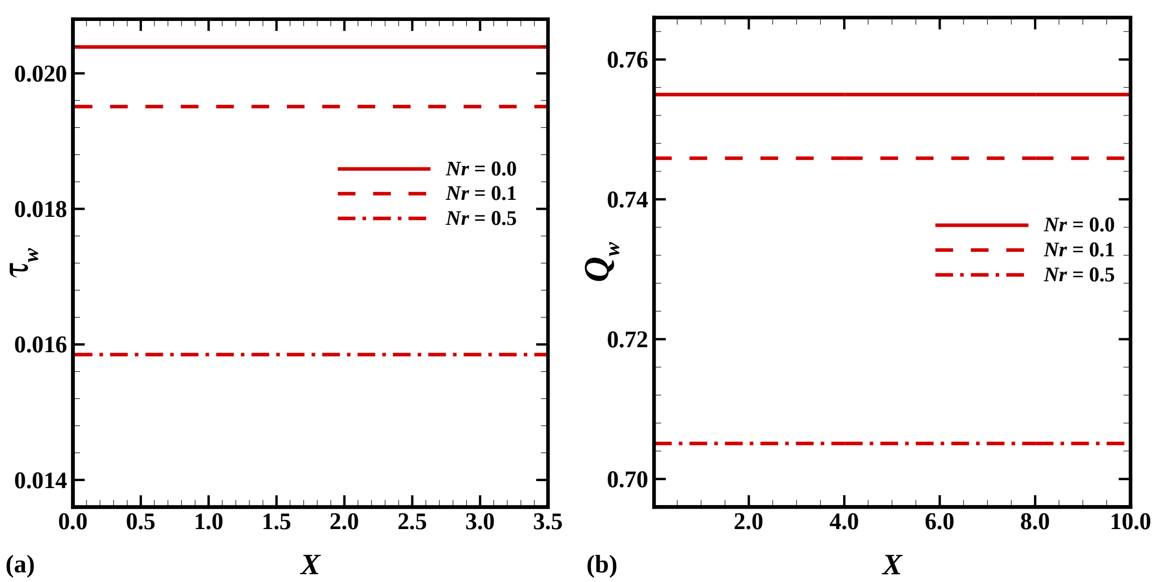


Fig. 5(a) Skin friction and (b) Rate of heat transfer coefficients for $Nr = 0.0, 0.1, 0.5$ **while** $\mathbf{Pr} = 7.0$, $n_\mu = 1.04$, $n_\lambda = 1.185$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Rb = 2.0$, $N_A = N_B = 2.0$, $\lambda = 2.0$ **and** $\Omega = 0.1$.

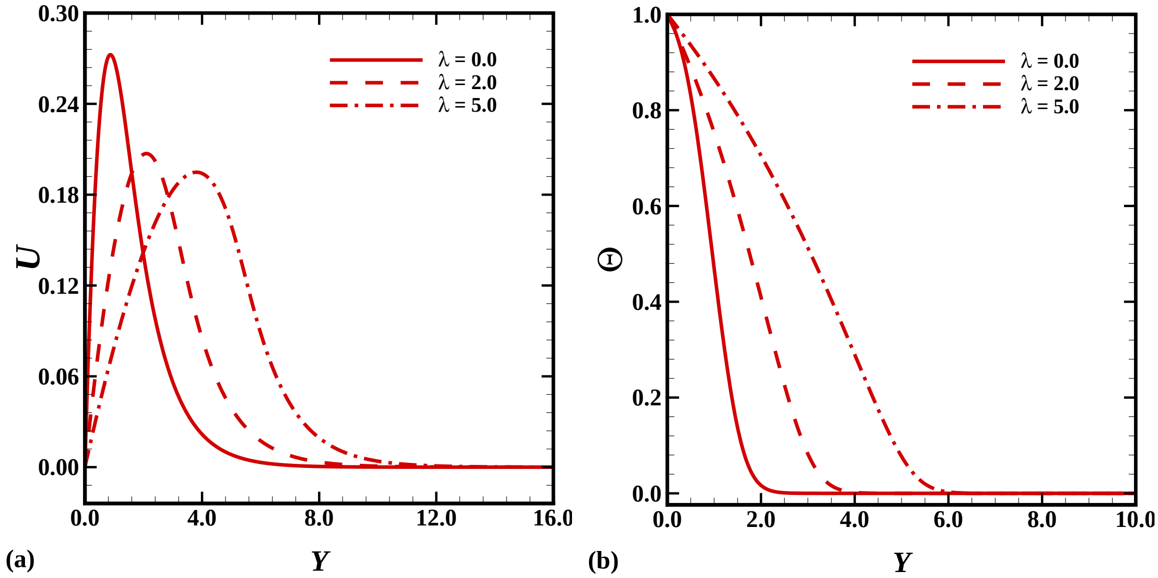


Fig. 6(a) Velocity and (b) Temperature profiles for $\lambda = 0.0, 2.0, 5.0$ **while** **Pr** $= 7.0$, $n_\mu = 1.04$, $n_\lambda = 1.185$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Nr = Rb = 0.1$, $N_A = N_B = 2.0$, $\Omega = 0.1$ **and** $X = 10.0$.

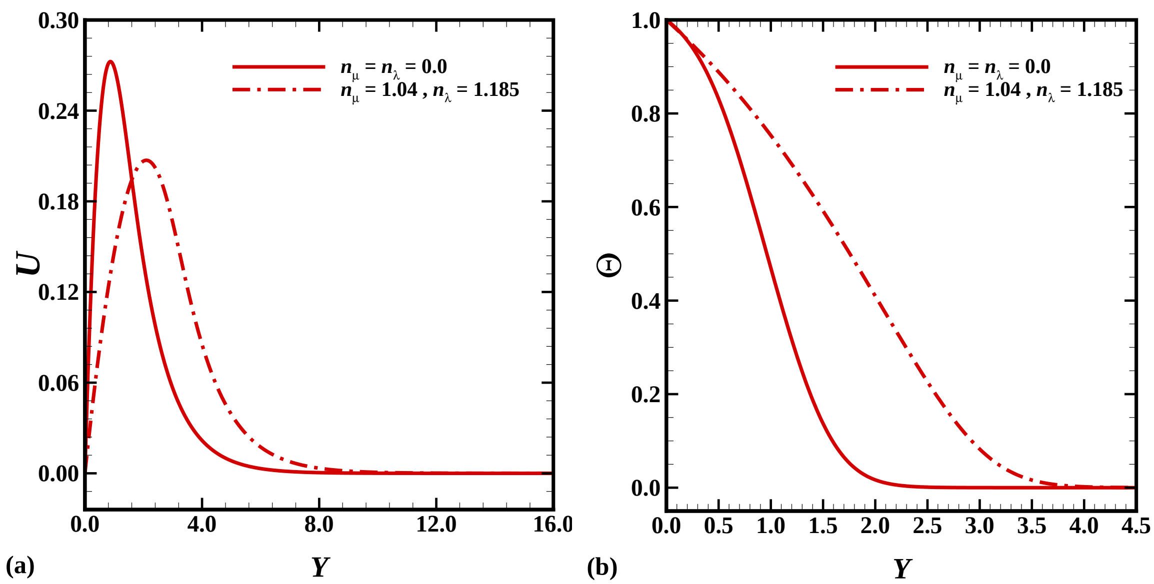
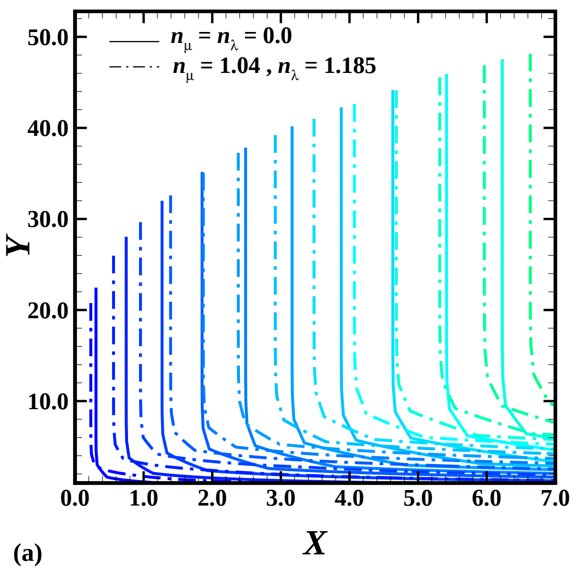


Fig. 7(a) Velocity and (b) Temperature profiles for $n_\mu = 0.0, 1.04$, $n_\lambda = 0.0, 1.185$, $\lambda = 0.0, 2.0$ **while Pr** $= 7.0$, $Pe = 0.1$, $Lb = Ln = 10.0$, $Nr = Rb = 0.1$, $N_A = N_B = 2.0$, $\Omega = 0.1$ **and** $X = 10.0$.



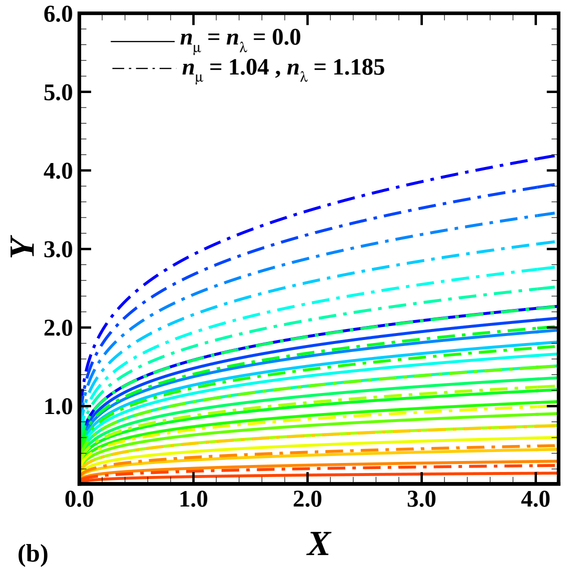


Fig. 8(a) Streamlines and (b) Isotherms for water with $n_\mu = 0.0, 1.04$**,** $n_\lambda = 0.0, 1.185$**,** $\lambda = 0.0, 2.0$ **while** $\mathbf{Pr} = 7.0$**,** $Pe = 0.1$**,** $Lb = Ln = 10.0$**,** $Nr = Rb = 0.1$**,** $N_A = N_B = 2.0$**, and** $\Omega = 0.1$**.**