UNIVERSITÄT DORTMUND

Numerical benchmarking of fluid-structure interaction between elastic object and laminar incompressible flow

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Key questions

☞ Accurate and robust description of the **interaction mechanisms** w.r.t. highly dynamical and nonlinear behaviour and significant geometry changes? That includes:

- Quality of different discretization techniques (FEM, FV, FD, LBM, resp., beam, shell, volume elements) for FSI?
- Robustness and numerical efficiency of the integrated solver components?

☞ **Evaluation of partitioned approaches vs. monolithic schemes?**

1st step: *Identification of appropriate FSI setting for numerical benchmarking*

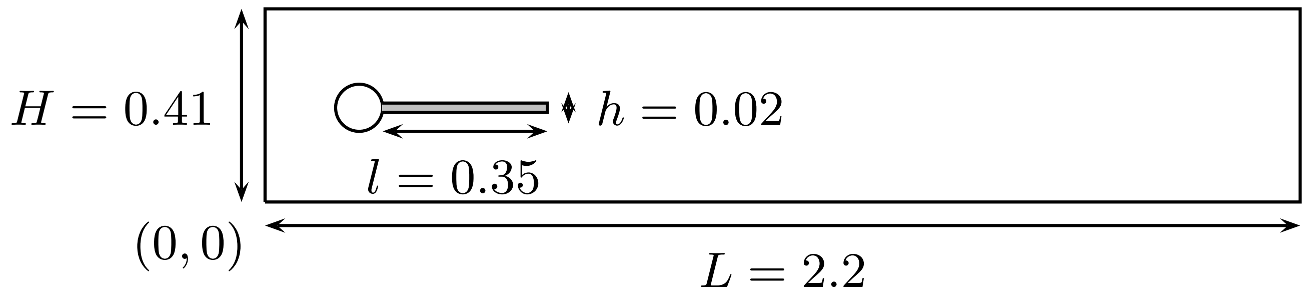
2nd step: *FSI benchmark setting due to experimental studies*

Requirements for numerical FSI benchmarking

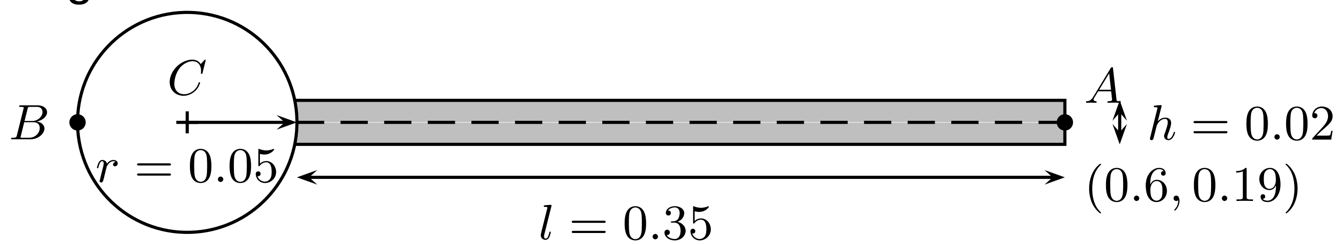
- mainly based on the successful DFG *flow around cylinder* benchmark
- *realistic materials*
  - **incompressible Newtonian fluid**, laminar flow regime
  - **elastic solid**, large deformations
- *comparative evaluation*
  - setup with periodical oscillations
  - non-graphically based quantities
- *computable configurations*
  - laminar flow
  - reasonable aspect ratios
  - simple geometry (2D)

Computational domain

- domain dimensions

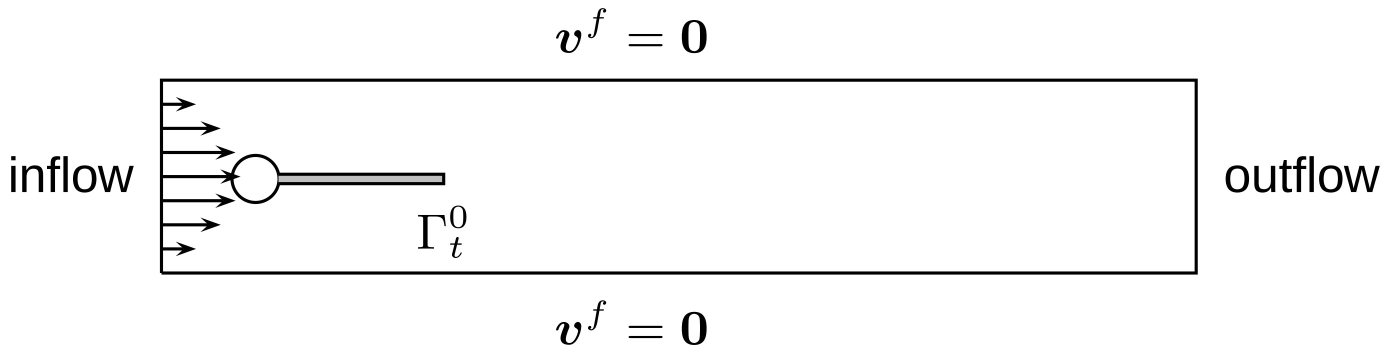


- detail of the submerged structure



- $A(t = 0) = (0.6, 0.2), \quad B = (0.15, 0.2), \quad C = (0.2, 0.2)$

Boundary and initial conditions



inflow parabolic velocity profile is prescribed at the left end of the channel

outflow condition can be chosen by the user, assuming zero reference pressure (*stress free* or *do nothing*)

interface condition on Γ_t^0 is $\boldsymbol{v}^f = \boldsymbol{v}^s$ and $\boldsymbol{\sigma}^f \boldsymbol{n} = \boldsymbol{\sigma}^s \boldsymbol{n}$

otherwise the *no-slip* condition is prescribed for the fluid on the other boundary parts. i.e. top and bottom wall and cylinder

initial zero velocity in the fluid and no deformation of the structure + smooth increase of the inflow profile

Fluid and structure properties

☞ **Incompressible** fluid with density ϱ^f

$$\left.\begin{aligned} \varrho^f \frac{\partial \boldsymbol{v}^f}{\partial t} + \varrho^f (\nabla \boldsymbol{v}^f) \boldsymbol{v}^f &= \operatorname{div} \boldsymbol{\sigma}^f \\ \operatorname{div} \boldsymbol{v}^f &= 0 \end{aligned}\right. \qquad \text{in } \Omega_t^f$$

$$\boldsymbol{\sigma}^f = -p^f \mathbf{I} + \varrho^f \nu^f (\nabla \boldsymbol{v}^f + \nabla \boldsymbol{v}^{f\,\mathrm{T}})$$

☞ Elastic material with density ϱ^s, $\mathbf{F} = \mathbf{I} + \nabla \boldsymbol{u}^s$, $J = \det \mathbf{F}$: **St. Venant – Kirchhoff** material

$$\varrho^s \frac{\partial^2 \boldsymbol{u}^s}{\partial t^2} = \operatorname{div}(\boldsymbol{\sigma}^s \mathbf{F}^{-\mathrm{T}}) \qquad \text{in } \Omega^s$$

$$\begin{aligned} \boldsymbol{\sigma}^s &= \frac{1}{J} \mathbf{F} (\lambda^s (\operatorname{tr} \mathbf{E}) \mathbf{I} + 2\mu^s \mathbf{E}) \mathbf{F}^{\mathrm{T}} \\ \mathbf{E} &= \frac{1}{2} (\mathbf{F}^{\mathrm{T}} \mathbf{F} - \mathbf{I}) \end{aligned}$$

Suggested material parameters

solid

ϱ^s density
ν^s Poisson ratio
μ^s shear modulus

fluid

ϱ^f density
ν^f kinematic viscosity

parameter	polybutadiene & glycerine	**polypropylene** & **glycerine**
ϱ^s $[10^3 \frac{\text{kg}}{\text{m}^3}]$	0.91	1.1
ν^s	0.5	0.42
μ^s $[10^6 \frac{\text{kg}}{\text{ms}^2}]$	0.53	317
ϱ^f $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1.26	1.26
ν^f $[10^{-3} \frac{\text{m}^2}{\text{s}}]$	1.13	1.13

parameter	FSI1	FSI2	FSI3
ϱ^s $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1	1	1
ν^s	0.4	0.4	0.4
μ^s $[10^6 \frac{\text{kg}}{\text{ms}^2}]$	0.5	0.5	2.0
ϱ^f $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1	1	1
ν^f $[10^{-3} \frac{\text{m}^2}{\text{s}}]$	1	1	1
$\bar{U}$ $[\frac{\text{m}}{\text{s}}]$	0.2	1	2

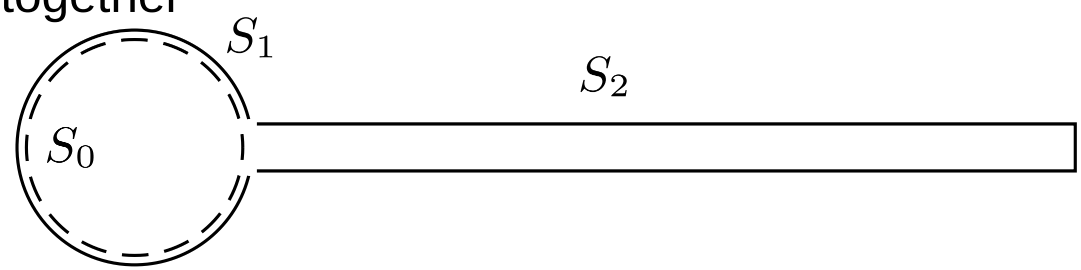
parameter	FSI1	FSI2	FSI3
$\beta = \frac{\varrho^s}{\varrho^f}$	1	1	1
ν^s	0.4	0.4	0.4
Ae $= \frac{E^s}{\varrho^f \bar{U}^2}$	3.5×10^4	1.4×10^3	1.4×10^3
Re $= \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U}$	0.2	1	2

Quantities of interest

- the position $A(t) = (x(t), y(t))$ of the end of the structure
- pressure difference between the points $A(t)$ and B

$$\Delta p^{AB}(t) = p^B(t) - p^{A(t)}(t)$$

- forces exerted by the fluid on the *whole* body, i.e. lift and drag forces acting on the cylinder and the structure together



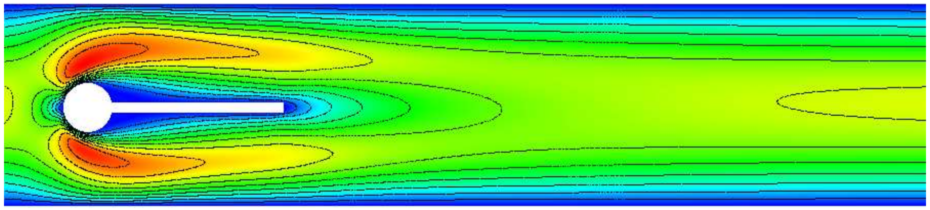
$$(F_D, F_L) = \int_S \boldsymbol{\sigma} \boldsymbol{n} dS = \int_{S_1} \boldsymbol{\sigma}^f \boldsymbol{n} dS + \int_{S_2} \boldsymbol{\sigma}^{f|s} \boldsymbol{n} dS = \int_{S_0} \boldsymbol{\sigma} \boldsymbol{n} dS$$

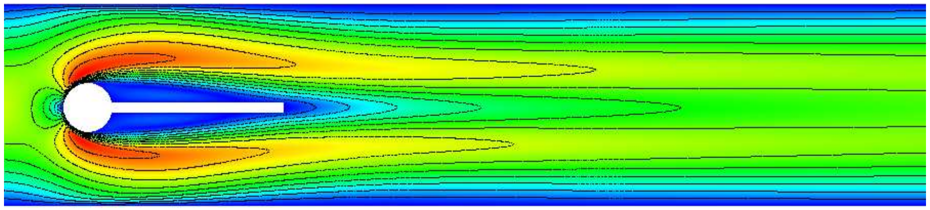
- frequency and maximum amplitude
- compare results for *one* full period and 3 different levels of spatial discretization h and 3 time step sizes Δt

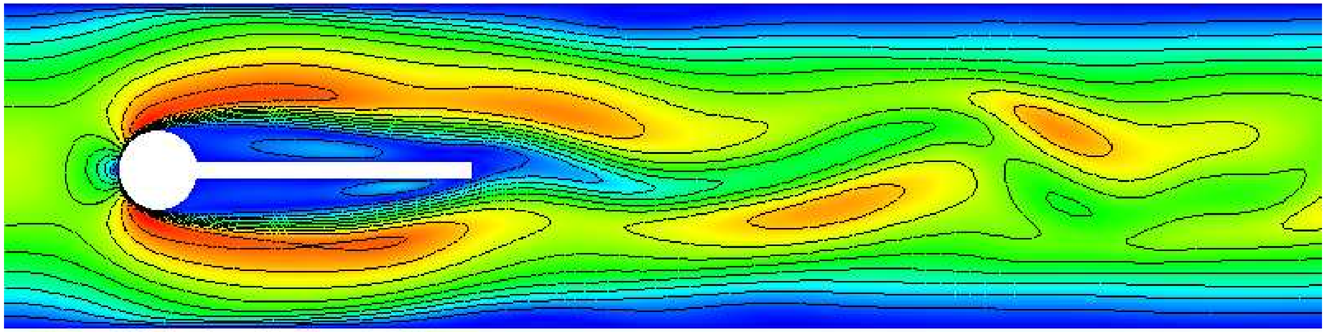
1.Step: CFD tests for validation

	CFD1	CFD2	CFD3
ϱ^f $[10^3 \frac{\mathrm{kg}}{\mathrm{m}^3}]$	1	1	1
ν^f $[10^{-3} \frac{\mathrm{m}^2}{\mathrm{s}}]$	1	1	1
$\bar{U}$ $[\frac{\mathrm{m}}{\mathrm{s}}]$	0.2	1	2
$\mathrm{Re} = \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U}$	0.2	1	2

test	drag	lift
CFD1	14.29	1.119
CFD2	136.7	10.53
CFD3	439.4 ± 5.618 $[4.395]$	-11.89 ± 437.8 $[4.395]$



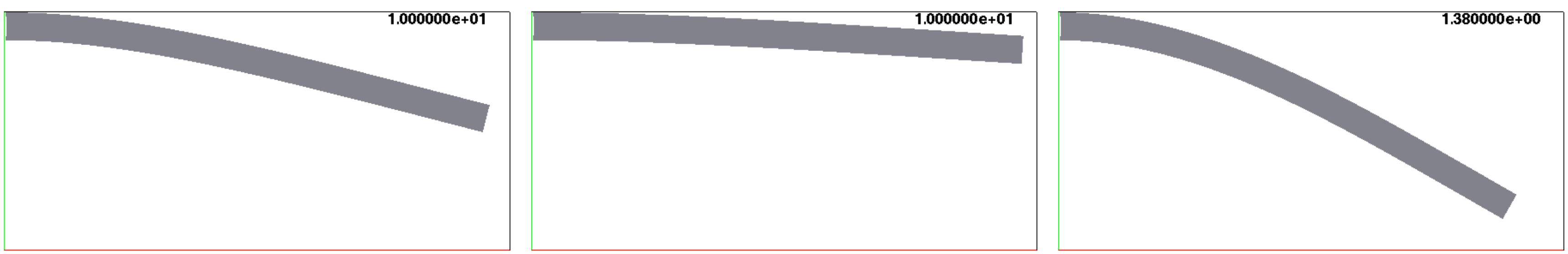




2.Step: CSM tests for validation

	CSM1	CSM2	CSM3
ϱ^s $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1	1	1
ν^s	0.4	0.4	0.4
μ^s $[10^6 \frac{\text{kg}}{\text{ms}^2}]$	0.5	2.0	0.5
g $[\frac{\text{m}}{\text{s}^2}]$	2	2	2
$\beta = \frac{\varrho^s}{\varrho^f}$	1	1	1
ν^s	0.4	0.4	0.4
E^s $[10^6 \frac{\text{kg}}{\text{ms}^2}]$	1.4	5.6	1.4
g	2	2	2

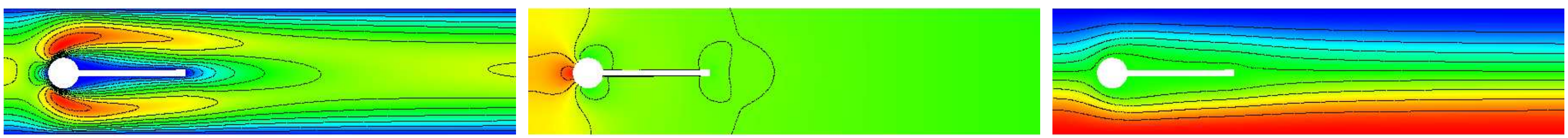
test	ux of A $[\times 10^{-3}\text{m}]$	uy of A $[\times 10^{-3}\text{m}]$
CSM1	-7.187	-66.10
CSM2	-0.4690	-16.97
CSM3	-14.305 ± 14.305 $[1.0995]$	-63.607 ± 65.160 $[1.0995]$



FSI1: steady, small deformations

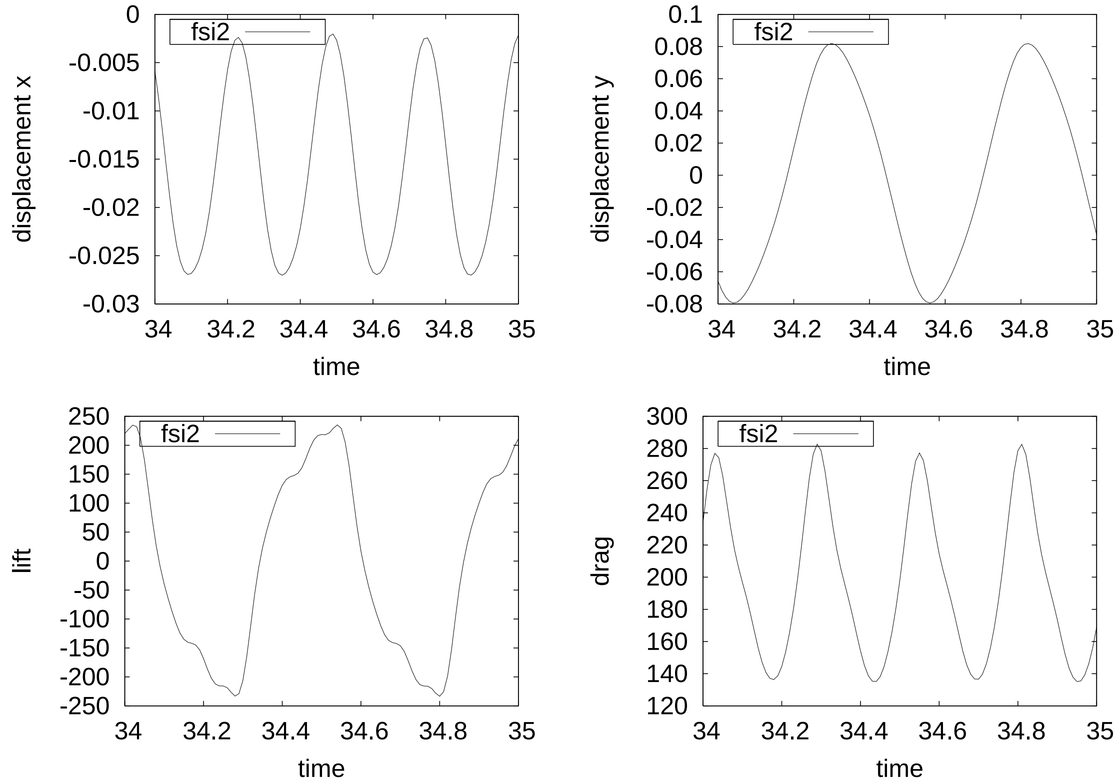
parameter	FSI1	FSI2	FSI3
ϱ^s $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1	1	1
ν^s	0.4	0.4	0.4
μ^s $[10^6 \frac{\text{kg}}{\text{ms}^2}]$	0.5	0.5	2.0
ϱ^f $[10^3 \frac{\text{kg}}{\text{m}^3}]$	1	1	1
ν^f $[10^{-3} \frac{\text{m}^2}{\text{s}}]$	1	1	1
$\bar{U}$ $[\frac{\text{m}}{\text{s}}]$	0.2	1	2

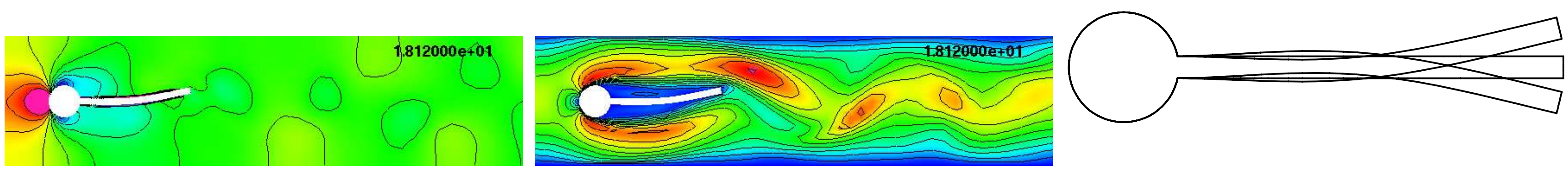
parameter	FSI1	FSI2	FSI3
$\beta = \frac{\varrho^s}{\varrho^f}$	1	1	1
ν^s	0.4	0.4	0.4
Ae $= \frac{E^s}{\varrho^f \bar{U}^2}$	3.5×10^4	1.4×10^3	1.4×10^3
Re $= \frac{\bar{U} d}{\nu^f}$	20	100	200
$\bar{U}$	0.2	1	2



	ux of A $[\times 10^{-3}\text{m}]$	uy of A $[\times 10^{-3}\text{m}]$	drag	lift
FSI1	0.0227	0.8209	14.295	0.7638

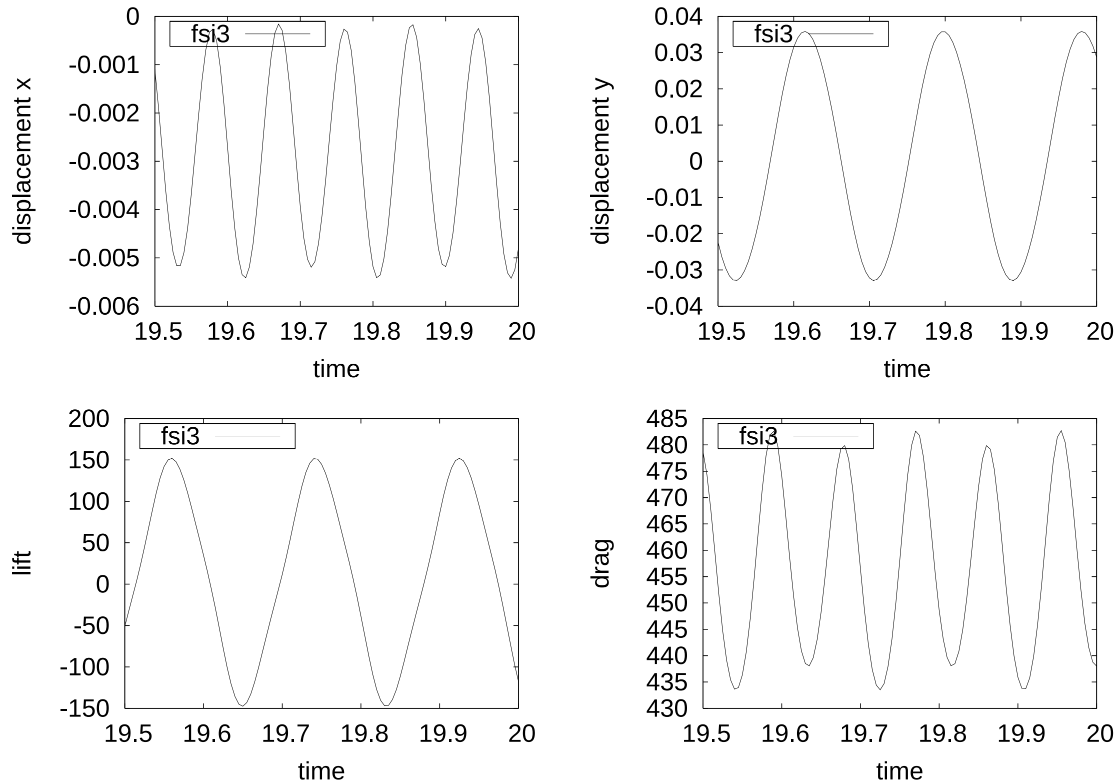
FSI2: large deformations, periodical oscillations

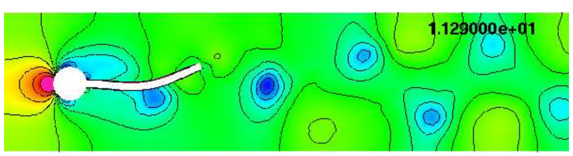


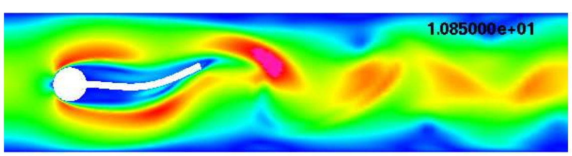


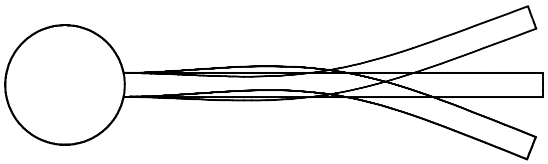
test	ux of A $[\times 10^{-3}$m$]$	uy of A $[\times 10^{-3}$m$]$	drag	lift
FSI2	$-14.58 \pm 12.44[3.8]$	$1.23 \pm 80.6[2.0]$	$208.83 \pm 73.75[3.8]$	$0.88 \pm 234.2[2.0]$

FSI3: large deformations, complex oscillations









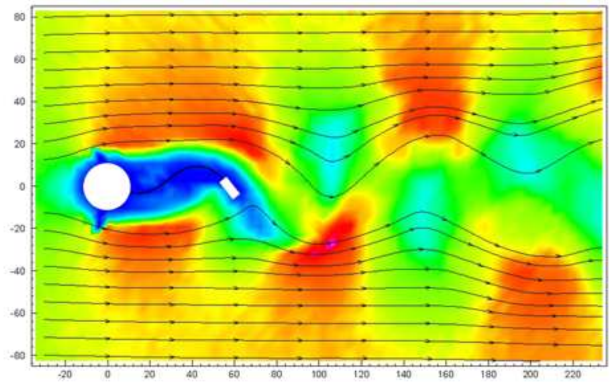
test	ux of A [$\times 10^{-3}$m]	uy of A [$\times 10^{-3}$m]	drag	lift
FSI3	$-2.69 \pm 2.53[10.9]$	$1.48 \pm 34.38[5.3]$	$457.3 \pm 22.66[10.9]$	$2.22 \pm 149.78[5.3]$

Current status

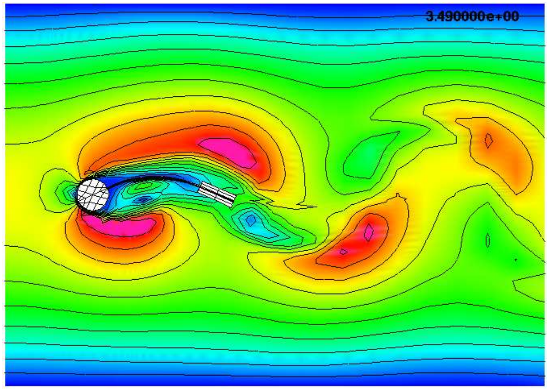
- ☞ Verification of reference values is almost finished ($\rightarrow$ new version)
- ☞ Subtests for validating CFD and CSM components are available:
  - CSM1-3: "OK"
  - CFD1: "easy" $\rightarrow Re = 20$
  - CFD2: (also) "easy" $\rightarrow Re = 100$
  - CFD3: "non-trivial" $\rightarrow Re = 200$
- ☞ FSI settings with desired properties:
  - FSI1: "simple" $\rightarrow$ for validation only
  - FSI3: "hard" $\rightarrow$ due to CFD3
  - FSI2: fully oscillating while CFD2 ($\approx$ same Re number!) is steady
    $\Rightarrow$ **Excellent check for interaction mechanisms**
- ☞ Evaluation and comparison of mathematical and algorithmic components - *everybody is invited to participate.*

Benchmarking of experimental data

- Flustruc experiment, Erlangen



experiment



computation

- First computational results and tasks