Tiling theorems and application to random Schrödinger operators with Gaussian random potential

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(joint work with Ivan Veselić)

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The first part of the talk is devoted to the study of tiling theorems of $\mathbb{R}^d$. The problem here is to identify a class of functions $\mathscr{F} \subset \mathscr{L}^1(\mathbb{R}^d)$, such that for all $f \in \mathscr{F}$ there is a signed Borel measure μ_f on $\mathbb{R}^d$, such that

$$\forall x \in \mathbb{R}^d\colon \quad \int_{\mathbb{R}^d} f(x-y)\mu_f(\mathrm{d}y) = 1.$$

Our main theorem provides a precise description of the measure μ_f for exponentially decaying, but arbitrarily sign-changing, functions f.

In the second part of the talk we apply this tiling theorems to random Schrödinger operators. Here we study a family of operators in $L^2(\mathbb{R}^d)$ of the type

$$H_\omega = -\Delta + V_\omega, \quad \omega \in \Omega,$$

where Δ denotes the Laplace operator and $V : \Omega \times \mathbb{R}^d \to \mathbb{R}$ is a stationary jointly measurable Gaussian field on a complete probability space $(\Omega, \mathscr{A}, \mathbb{P})$. If the covariance function decays exponentially (but may change its sign arbitrary) we prove a Wegner estimate for finite volume restrictions $H_{\omega,L}$ of H_ω to a cube of side length $L > 0$ with Dirichlet boundary conditions. This is an upper bound on the expected number of eigenvalues of $H_{\omega,L}$ within a bounded interval $I \subset \mathbb{R}$.