Post-Quantum Cryptography

P. Felke

26th of October
2023

How it all started

An:

E. Becker
Ralph Berr
Peter Caspers
Patrick Felke
Hagen Hannemann
Boris Hemkemeier
Helma Lettau
Carsten Moeller
Michael Niermann
Axel Pawellek
Frank Vallentin

Liebe Kryptographie-Seminar Interessenten,

Herr Becker und ich haben einen ersten Vorschlag fuer den Ablauf des Blockseminars "Kryptographie und Algebra" fertiggestellt.

TERMIN:
Preferenz:
Fr, 13. Nov/Sa, 14. Nov.

Ausweichtermin:
Fr, 6. Nov./Sa, November
Wir bitten um Stellungnahmen zu diesem Terminvorschlag.

VORBESPRECHUNG:
Der Termin fuer Seminarvorbesprechung ist nach wie vor:
Mittwoch, 21.10.98. 10.15 Uhr im Seminarraum M/SR 911.

LEKTUERE:
Dem Seminar liegen im Wesentlichen 2 Quellen zu Grunde:
1) Neal Koblitz: Algebraic Aspects of Cryptography, Springer Verlag

2) Beutelsbacher, Schwenk, Wolfenstetter: Moderne Verfahren der Kryptographie, Vieweg Verlag

Als Lektuere fuer alle vor dem Seminar setzen wir die Kapitel 1 und 2 aus dem Buch von Koblitz voraus (sind sehr gut zu lesen). Diese werden als Fotokopien bei Frau Jahn Anfang Oktober erhaeltlich sein. Das Buch von Koblitz ist leider teuer. Wir sehen zu es bald am Lehrstuhl zur Verfuegung zu haben.

Symmetric Cryptography

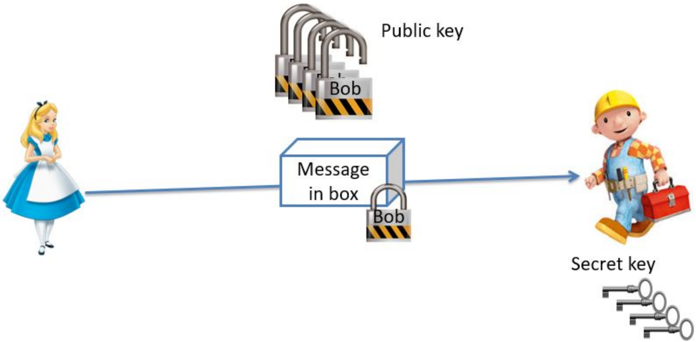
- Symmetric cryptography is a tool developed to ensure the confidentiality of a message.
- Alice encrypts a secret message with an encryption algorithm E and key k_{AB}. Bob decrypts the ciphertext by using a decryption algorithm D together with the same k_{AB}.
- An attacker with access to the channel should not be able to understand the communication.
- The key must be transmitted via a secure channel (out-of-band) between Alice and Bob.

Public-Key Cryptography

Public-key cryptography gives positive answers to the following questions:

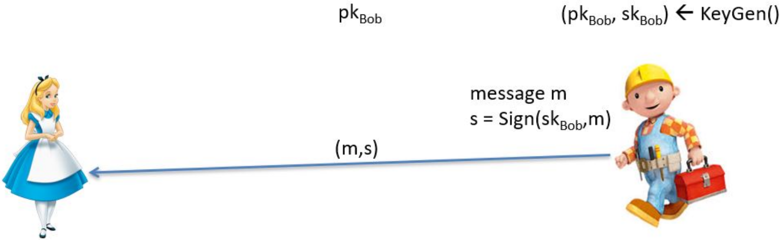
- Can two people who have never met have a private conversation?
- Is it possible to digitally sign documents?

Public-Key Encryption



- This is achieved by introducing cryptosystems using a pair of keys.
- Alice encrypts a message for Bob with Bob's public key pk_{Bob}.
- Bob decrypts the message with his secret key sk_{Bob}.
- pk_{Bob} can be transmitted over an insecure channel.
sk_{Bob} has to be stored securely.

Digital Signatures



- Bob signs a message for Alice with his secret key sk_{Bob}.
- Alice verifies the received signature with Bob's pk_{Bob}.

☞The famous RSA public-key cryptosystem can be easily turned into a signature algorithm. This also explains why it is so widespread nowadays.

☞The result of these positive answers is that

Cryptography is Ubiquitious

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Signal

It is deployed in

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It is deployed in

- messenger services,
- electronic commerce,
- automotive industry,
- cloud computing,
- ⋮

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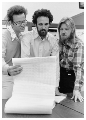

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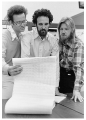

- Elliptic curve cryptography (ECC) is based on the special case of the elliptic curve discrete logarithm, $Q = sP, \log_P(Q) = s$.

These problems allow systems of small key sizes.

Practical Key Sizes

RSA bit size of modulus n		Dlog bit size of prime field $\mathbb{F}_p$		ECC bit size of field $\mathbb{F}_n$
2800	$\sim$	2800	$\sim$	240
3000	$\sim$	3000	$\sim$	250

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Technical guideline TR-02102-1, Version 2023-1 from the Federal Office for Information Security (FOIS).

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Practical Security (informal)

A cryptosystem is practical secure if the best known algorithm for breaking it requires (almost for sure) an unreasonable amount of time or memory using available computing power.

For the systems above the best known algorithms require to solve the underlying hard problem.

What are hard Problems?

James L. Massey:“A hard problem is a problem nobody works on.”

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- NSA has spent 85M$ on research to build a quantum computer.
- After the disclosure the National Institute for Standards and Technology (NIST, US pendant to FOIS) initiated the PQC competition.

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☞Symmetric cryptography remains secure when employed with larger but still moderate sized keys. These schemes remain practical.

The NIST Post-Quantum Cryptography Competition

https://csrc.nist.gov/Projects/Post-Quantum-Cryptography

- Start: 2016
- End: 2022, Draft standards until 2024.
- 2022: Round 4 submissions for backup candidates.

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Secure means again practical secure.

The new Candidates

The submitted candidates are based on the following hard problems with respect to post-quantum cryptography.

- Lattice-based crypto (e.g. hardness of finding short vectors).
- Code-based crypto (hardness of decoding a random code).
- Multivariate crypto (hardness of solving a random system of quadratic equations).

- Most designs discussed in this competition are not new. Due to their large key sizes they were rarely employed in practice before this competition.
  Up to 1 Mb for a security level of e.g. 2048 bit ≈ 200 byte RSA.

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  A good reason to have a look at these cryptosystems.

Introduction

The PQC Competition

Multivariate Cryptography

Finalists of PQC Competition

Going Back to the 80s

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- FOIS planned in the 90th to implement a variant in devices employed for national security.

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1. A finite Field $\mathbb{F}_q$, $q := 2^m$, a field extension of $\mathbb{F}_{q^n}$ of degree n, an $\mathbb{F}_q$-basis $\mathcal{B} = \{\beta_1, \ldots, \beta_n\}$ of $\mathbb{F}_{q^n}$.

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2. A $0 \leq \theta \leq n-1$, s.t. the power function $\pi(X) := X^{q^\theta+1}$ is bijective, i.e. $\gcd(q^\theta+1, q^n-1) = 1$.
3. The inverse mapping, which is the power mapping $\pi^{-1}(X) = X^h$ with $h(q^\theta+1) = 1 \bmod q^n - 1$.

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Secret key:

1. Affine Transformations $S = Ax + d$, $T = Bx + e$, $A, B \in GL(n, \mathbb{F}_q)$, und $d, e \in \mathbb{F}_q^n$.

☞Condition 2 requires q^n to be even. It is easy to find proper n such that condition 2 can be fulfilled.

Construction of the Public-Key

Compute the multivariate representation of $\pi(X) = X^{q^\theta+1}$ with respect to $\mathcal{B}$.

It is $\beta_i^{q^\theta} = \sum_{l=1}^{n} p_{il}^{(\theta)} \beta_l$ and $\beta_i \beta_j = \sum_{l=1}^{n} m_{ijl} \beta_l, p_{il}^{(\theta)}, m_{ijl} \in \mathbb{F}_q$.

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$\sum_{1\le i\le j\le n} a_{ij}^{(l)} u_iu_j =: f_l(u_1,\dots,u_n) \Rightarrow \pi(\mathbf{u}) = \sum_{l=1}^n f_i(u_1,\dots,u_n)\beta_l$.

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The quadratic polynomials $p_1,\dots,p_n$ constitute the public key.

The computations are mod $x_1^q + x_1,\dots,x_n^q + x_n$.

Encryption/Decryption with C^*

Encryption (public):

$$m \in \mathbb{F}_q^n \xrightarrow{c=(p_1(m),\ldots,p_n(m))^t=T\circ\boldsymbol{\pi}\circ S(m)} c \in \mathbb{F}_q^n$$

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Decryption (secret):

$$\begin{array}{ccccccc}
m & \xleftarrow{S^{-1}} & \mathbb{F}_q^n & & \mathbb{F}_q^n & \xleftarrow{T^{-1}} & c \\
 & & \uparrow \Phi_{\mathcal{B}}^{-1} & & \downarrow \Phi_{\mathcal{B}} & & \\
 & & \mathbb{F}_{q^n} & \xleftarrow{\pi(X)^{-1}=X^h} & \mathbb{F}_{q^n} & &
\end{array}$$

$$\Phi_{\mathcal{B}}(v) := \sum_{i=1}^n v_i\beta_i$$

Encryption/Decryption

- Alice encrypts a message $m = (m_1, \ldots, m_n)$ by computing

$$
\begin{aligned}
c_1 &= p_1(m_1, \ldots, m_n) \\
&\vdots \\
c_n &= p_n(m_1, \ldots, m_n)
\end{aligned}
$$

- Bob decrypts the ciphertext $c = (c_1, \ldots, c_n)$ by computing
  1. $v = T^{-1}(c)$
  2. $\pi^{-1}(\sum v_i \beta_i) = (\mathbf{v})^h = \mathbf{u} = \sum u_i \beta_i \Rightarrow u = (u_1, \ldots, u_n)$
  3. $m = S^{-1}(u)$

Security

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Malicious Eve (attacker) faces the problem to solve the following system of quadratic equations

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\begin{aligned}
c_1 &= p_1(x_1, \ldots, x_n) \\
&\vdots \\
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which has the unique solution $(m_1, \ldots, m_n)^t$ in $\mathbb{F}_q^n$.

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Solving a system of m quadratic equations in n variables is NP-hard with respect to worst case complexity.

The decomposition problem, i.e. recovering the secret key S, T is supposed to be even harder.
Interpolating the inverse mapping is also infeasible.

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- This yields at least $k \geq n$ multivariate relations
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  fulfilled for all plaintext-ciphertext pairs (m, c).
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  These can be easily computed from the public key.
- Both proved that plugging in an intercepted ciphertext c yields a system of linear equations $r_l(x,c) = 0, 1 \leq l \leq k$ with a solution space of dimension $\leq \frac{n}{3}$.
  ☠ The plaintext can be recovered efficiently for all practical key sizes.
- The attack does not recover the secret key.

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3. Security evaluations are more complicated.

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☞ Many new approaches followed.

Current Status (excerpt)

- C^* (Imai-Matsumoto, Eurocrypt'88):
  broken (Dobbertin '93 (classified), Patarin, Crypto'95).
- Quartz (Patarin et al., Cryptographers Track RSA 2001):
  broken (Courtois, Daum, Felke , PKC 2003).
- SFLASH (Patatrin et al., 2001):
  broken (V. Dubois, P.A. Fouque, Crypto 2007)
- HFE and variants with branches (Patarin, Eurocrypt 1996):
  broken (L. Bettale et al., DCC 2013, P. Felke, WCC 2006).
- EFLASH (Cartor at al., SAC'18):
  broken (Øygarden, Felke et al., Cryptographers Track RSA 2020).
- Dob (Patarin et al., IACR Cryptol. ePrint Arch., 2018):
  broken (Øygarden, Felke et al., PKC 2021/J. of Crypt. (wip)).
- GeMSS (Faugere et al, submission to NIST PQC comp.):
  broken (Chengdong et al, Crypto 2021).
- Rainbow (Ding et al., NIST PQC candidate):
  broken (W. Beullens, Crypto, 2022).

Introduction

The PQC Competition

Multivariate Cryptography

Finalists of PQC Competition

Finalists and 4th Round Candidates

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On the 7th of July 2022 NIST announced after 3 rounds the algorithms to be standardized:

- public-key encryption: CRYSTALS-Kyber (lattice-based),
- digital signatures : CRYSTALS-Dilithium, FALCON, SPHINCS+ (all lattice-based).

4th round candidates (to have non-lattice-based alternatives):

- public-key encryption: Classic McEliece, Bike and HQC (code-based).

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This raises the question . . .

. . . should one start to implement the candidates?

- Companies like Google, Microsoft etc. started to employ and promote usage of PQC.
  Thus customers will ask for it in other branches.
- FOIS gives the following advice (technical guidelines 2021-1): Employ Classic McElice (as cryptanalysed since 1978) or another candidate in combination with a classic standard like ECC to e.g. derive two separate symmetric keys and from those a single key.
  The details are given in the guideline.
- Sooner or later PQC will be compulsory to fulfil certain guidelines.
- The keys are much bigger. Up to 1 Mb in comparison to 3000 bit nowadays.

☞ Industry has to react now as changes later might be impossible, e.g. in a hardware solutions or devices with too less memory.

A big challenge . . .

Security Issues

- History has shown that most of the cyberattacks against security solutions do not break the underlying crypto. It is exploited how the crypto is implemented or employed.
- The transition to PQC requires considerable changes in software and hardware.
- It is expected that these will open the door for new cyberattacks.

Thank you.
Any Questions?