Hyperbolic systems with variable multiplicities

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Abstract

We consider the Cauchy problem

$$\text{(CP)} \qquad \begin{cases} D_t u = A(t,x,D_x)u + B(t,x,D_x)u + f(t,x) & , \quad (t,x) \in [0,T] \times \mathbb{R}^n \\ u|_{t=0} = u_0(x) & , \quad x \in \mathbb{R}^n \end{cases} ,$$

where $A(t,x,D_x) = [A_{ij}(t,x,D_x)]_{i,j=1}^m$, is a matrix of first order pseudodifferential operators and $B(t,x,D_x) = [B_{ij}(t,x,D_x)]_{i,j=1}^m$ is a matrix of zero order pseudodifferential operators and we assume that (CP) is hyperbolic, i.e. all eigenvalues $\lambda_i(t,x,\xi)$, $i = 1,\dots,m$ of the matrix $A(t,x,\xi)$ are real. We consider the well-posedness of system (CP) in C^∞, Gevrey spaces $\gamma^{(s)}$, and Sobolev spaces without assuming that the multiplicities of the eigenvalues $\lambda_i(t,x,\xi)$, $i = 1,\dots,m$ stay constant. First, we will consider the t-dependent case and then include the spatial variable x. In the latter case, it will be crucial that $[B_{ij}(t,x,D_x)]_{i,j=1}^m$ has some structure with respect to the order of the operators $B_{ij}(t,x,D_x)$.

The first case, t-dependent coefficients and well-posedness in C^∞ and $\gamma^{(s)}$, is published in [1] and is based on Energy estimates.

The second case, involving coefficients depending on t and x, is ongoing work and based on the construction of representations of solutions to (CP) in terms of Fourier integral operators.

The talk is based on published and ongoing work together with Claudia Garetto (Loughborough University) and Michael Ruzhansky (Imperial College)

Bibliography

[1] Claudia Garetto and Christian Jäh, *Well-posedness of hyperbolic systems with multiplicities and smooth coefficients*, Mathematische Annalen, 2016, doi:10.1007/s00208-016-1436-8