Bsp.:

- $f: \mathbb{R} \to \mathbb{R}, \quad x \mapsto 2x+1$

$y = 2x+1$ ← höchstens dann sinnvoll, wenn $x, y$ reelle Zahlen ohne Einheiten sind

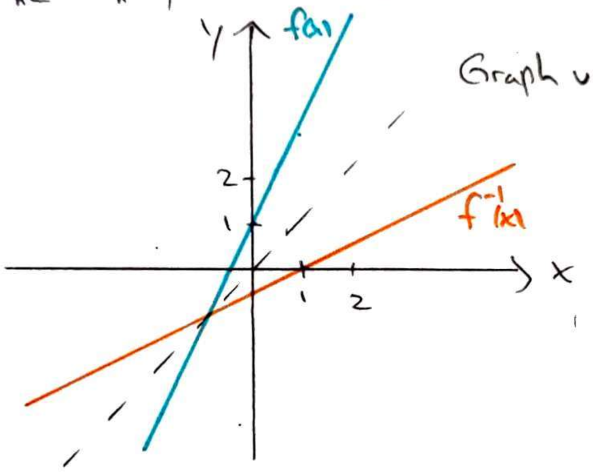
$x = 2y+1 \quad | -1$

$x-1 = 2y \quad | :2$

$y = \frac{1}{2}x - \frac{1}{2}$

sonst: $x = \frac{1}{2}y - \frac{1}{2} = f^{-1}(y)$

$f^{-1}: \mathbb{R} \to \mathbb{R}, \quad x \mapsto \frac{1}{2}x - \frac{1}{2}$




Graph von $f^{-1}$ = Graph von $f$ gespiegelt an der Winkelhalbierenden

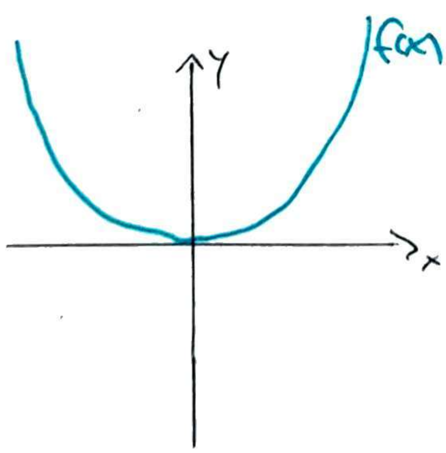
- $f: \mathbb{R} \to \mathbb{R}, \quad x \mapsto x^2$

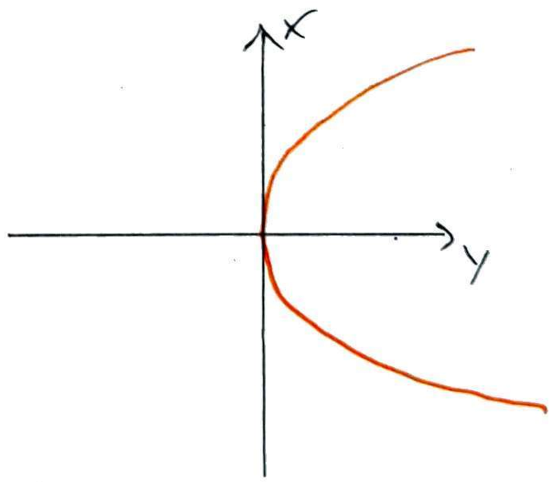
$y = x^2$

$x = \pm\sqrt{y}$

↖ 2. Problem: nur definiert für $y \in [0, \infty[$, nicht ganz $W = \mathbb{R}$

↖ 1. Problem: kein eindeutiger Funktionswert



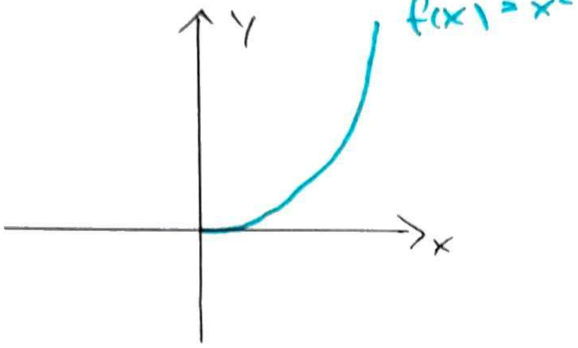



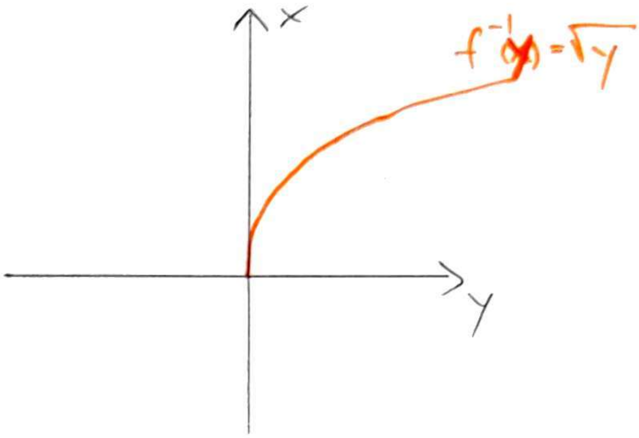

$f$ ist nicht umkehrbar / nicht invertierbar

- $f: [0,\infty[ \longrightarrow [0,\infty[, \quad x \longmapsto x^2$

  $y = x^2$

  $x = +\sqrt{y}$ (weil $x \in [0,\infty[$)

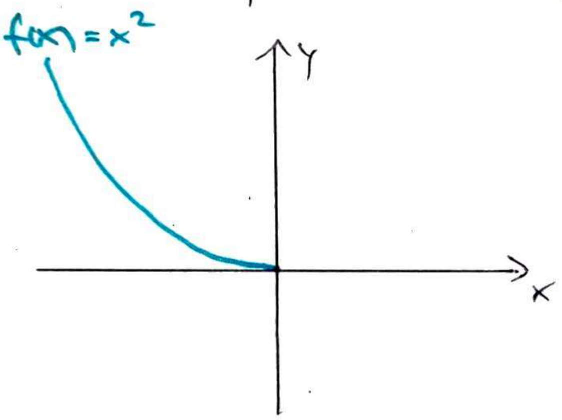


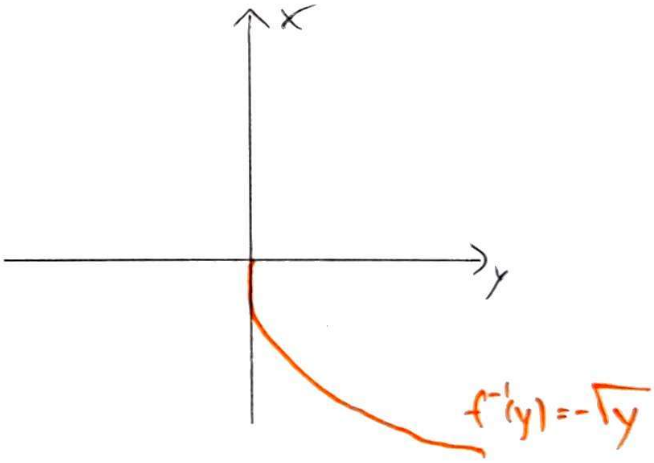




- $f: ]-\infty,0] \longrightarrow [0,\infty[, \quad x \longmapsto x^2$

  $y = x^2$

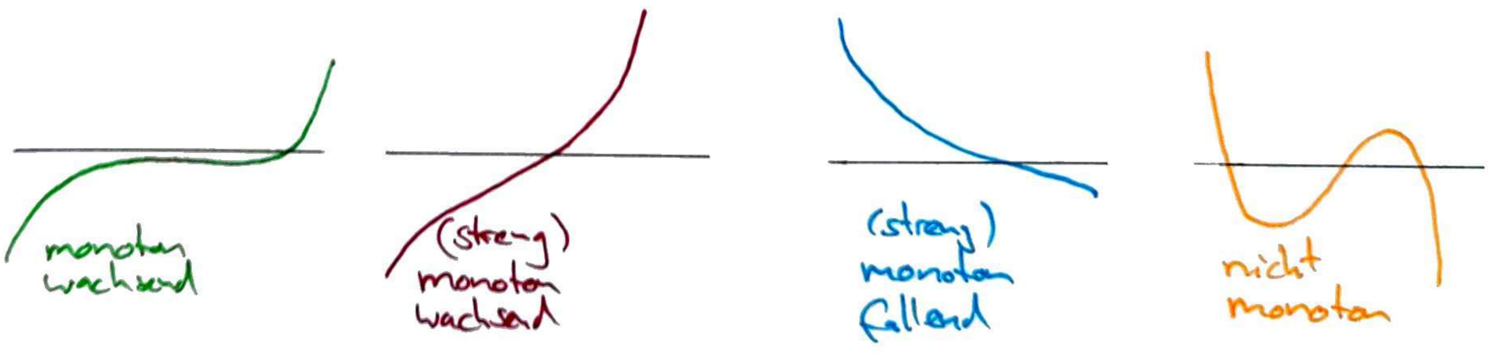
  $x = -\sqrt{y}$ (weil $x \in ]-\infty,0]$)



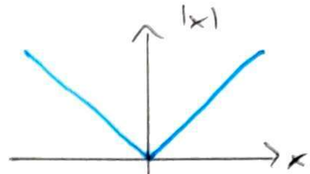

Def.: Mit einem Intervall $I \subset \mathbb{R}$ und $f: I \to \mathbb{R}$ heißt $f$

- monoton wachsend, wenn für $x_1 < x_2$ immer $f(x_1) \leq f(x_2)$ gilt
- streng monoton wachsend, —— " —— $f(x_1) < f(x_2)$ gilt
- monoton fallend, —— " —— $f(x_1) \geq f(x_2)$ gilt
- streng monoton fallend, —— " —— $f(x_1) > f(x_2)$ gilt

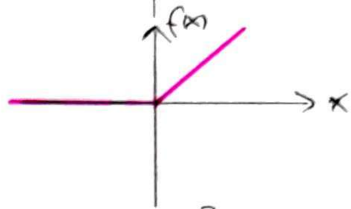


Bsp.: $f: \mathbb{R} \to \mathbb{R}$

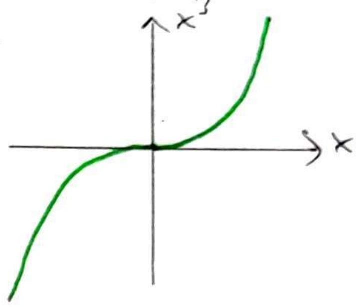
- $f(x) = 1$ monoton wachsend & fallend
- $f(x) = x$ (streng) monoton wachsend
- $f(x) = |x|$ nicht monoton



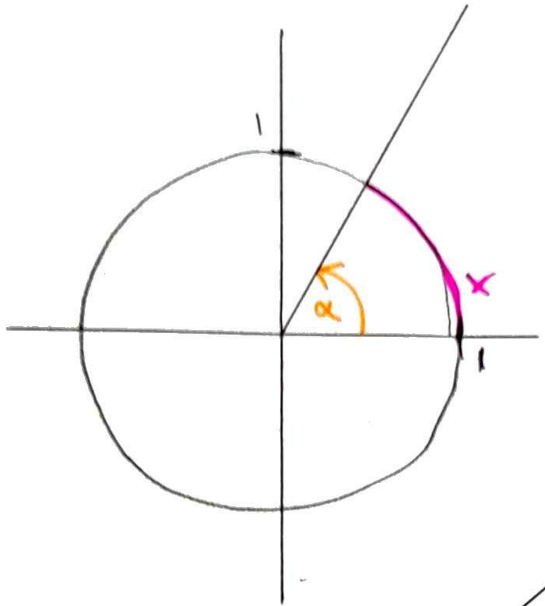

- $f(x) = \begin{cases} 0 & : x < 0 \\ x & : x \geq 0 \end{cases}$ monoton wachsend



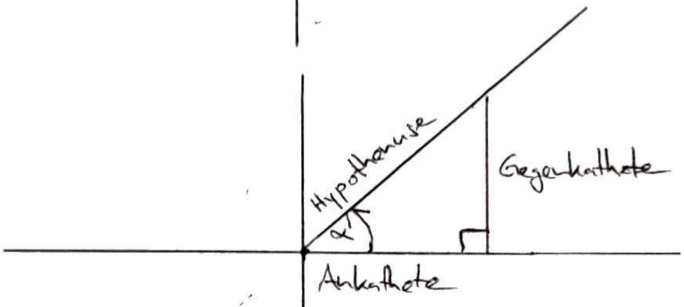

- $f(x) = x^2$ nicht monoton
- $f(x) = x^3$ (streng) monoton wachsend



# 5 Trigonometrie



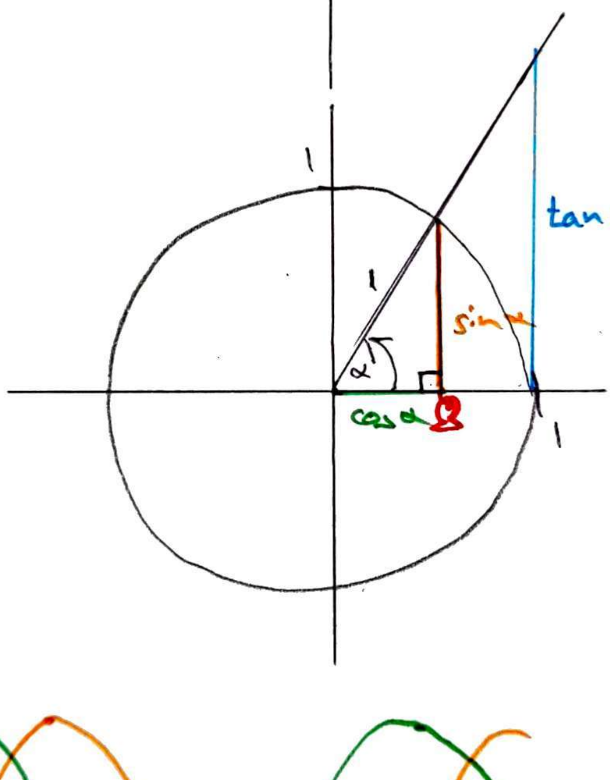

$$\frac{\alpha}{360^\circ} = \frac{x}{2\pi} = \frac{x}{\tau}$$

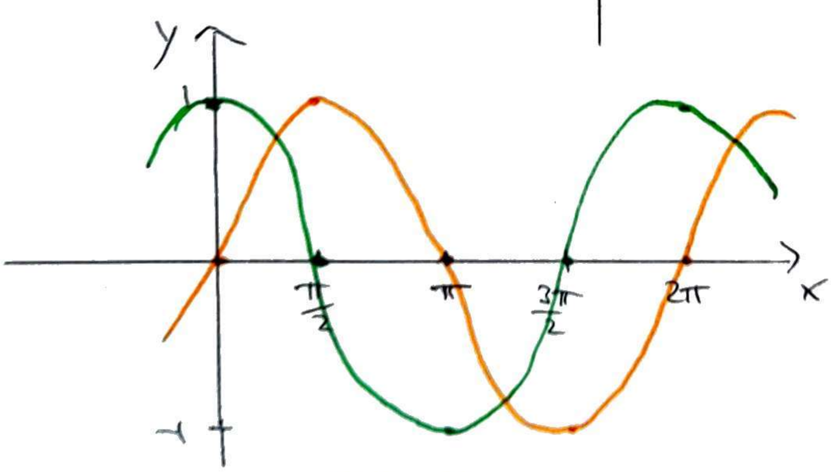


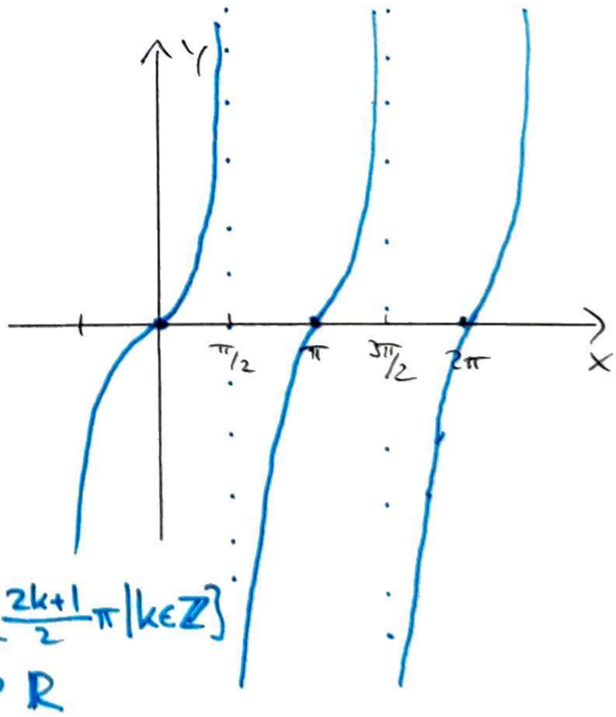

$$\sin\alpha = \frac{\text{Gegenkathete}}{\text{Hypothenuse}}$$

$$\cos\alpha = \frac{\text{Ankathete}}{\text{Hypothenuse}}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\text{Gegenkathete}}{\text{Ankathete}}$$




$\sin: \mathbb{R} \longrightarrow [-1, 1]$

$\cos: \mathbb{R} \longrightarrow [-1, 1]$

$\tan: \mathbb{R} \setminus \left\{ \frac{2k+1}{2}\pi \,\middle|\, k \in \mathbb{Z} \right\} \longrightarrow \mathbb{R}$